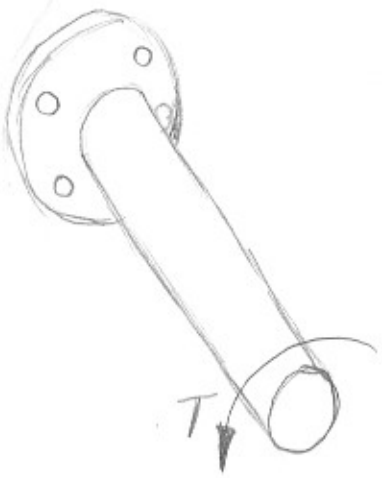
1.



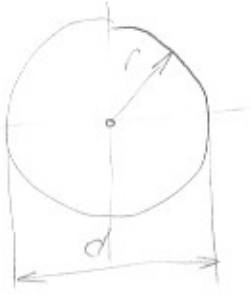
Odredit minimalan promjer punog vratila na koji djeluje moment uvijanja $T = 5\,kNm$

Dozvoljeni napon smicanja je $\tau_{dozv} = 65\,MPa$

Naprezanje smicanja u vratilu τ izazvan momentom uvijanja T je po formuli

$$\tau = \frac{T}{I_o} \cdot r \qquad \ldots (1)$$

U ovom problemu poznate su vrijednosti napona smicanja τ i momenta uvijanja T, a nepoznate su vrijednosti r i I_o. Nepoznate karakt. popr. presjeka r i I_o izrazićemo preko promjera d vratila:



$$r = \frac{d}{2} \qquad \ldots (2)$$

$$I_o = \frac{d^4 \pi}{32} \qquad (3)$$ - polarni moment inercije kružnog popr. presjeka

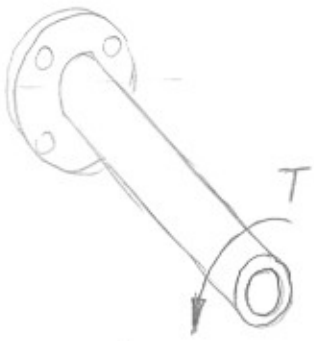
Formula (1) postaje

$$\frac{I_o}{r} = \frac{T}{\tau}$$

Odnos $\frac{I_o}{r} = \frac{\frac{d^4 \pi}{32}}{\frac{d}{2}} = \frac{d^3 \pi}{16} = W_o$ - Polarni otporni moment

$$\frac{d^3 \pi}{16} = \frac{T}{\tau} \qquad d = \sqrt[3]{\frac{16\,T}{\pi\,\tau}} = 73{,}2\,mm$$

Odrediti moment torzije T koji stvara maksimalan tangencijalni napon od 50MPa u prikazanom vratilu. Vanjski promjer vratila je 40mm, a debljina stijenke vratila je $\delta = 5mm$.

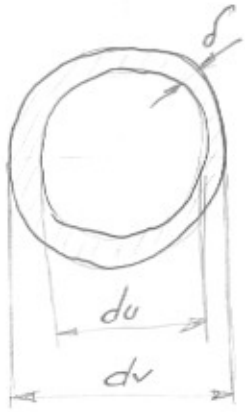


Tangencijalni napon torzije je: $\tau = \frac{T}{I_o} \cdot r$

Da bi koristili ovu formulu potrebno je još izračunati polarni moment inercije I_o:

$$I_o = \frac{\pi}{32}\left(d_v^4 - d_u^4\right)$$

d_v - vanjski promjer
d_u - unutrašnji promjer



$d_v = 40mm$
$\delta = 5mm$

$d_u = d_v - 2 \cdot \delta$
$= 40 - 2 \cdot 5$
$= 30mm$

$$I_o = \frac{\pi}{32}\left[(40mm)^4 - (30mm)^4\right] = 171\,805,85\,mm^4$$

$$\tau = \frac{T \cdot r}{I_0} \Rightarrow T = \frac{\tau \cdot I_0}{r}$$

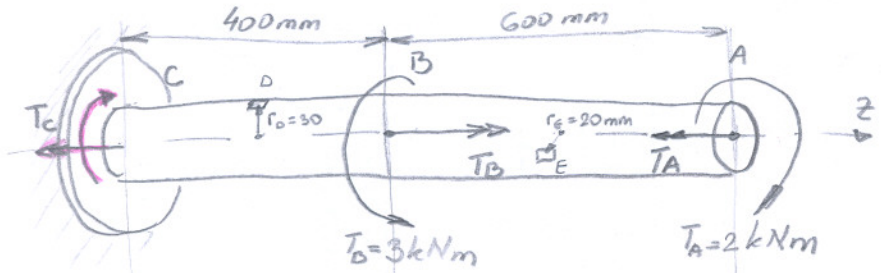
$$r = \frac{d_V}{2} = \frac{40\,mm}{2} = 20\,mm$$

$$T = \frac{50\,MPa \cdot 171\,805{,}85\,mm^4}{20\,mm}$$

$$T = 429\,514{,}5\,Nmm$$

$$T = 430\,Nm$$

Čelično vratilo na slici ima prečnik 60 mm. Odrediti ugao uvijanja kraja A vratila u odnosu na kraj C. Odrediti napone u elementima D i E.

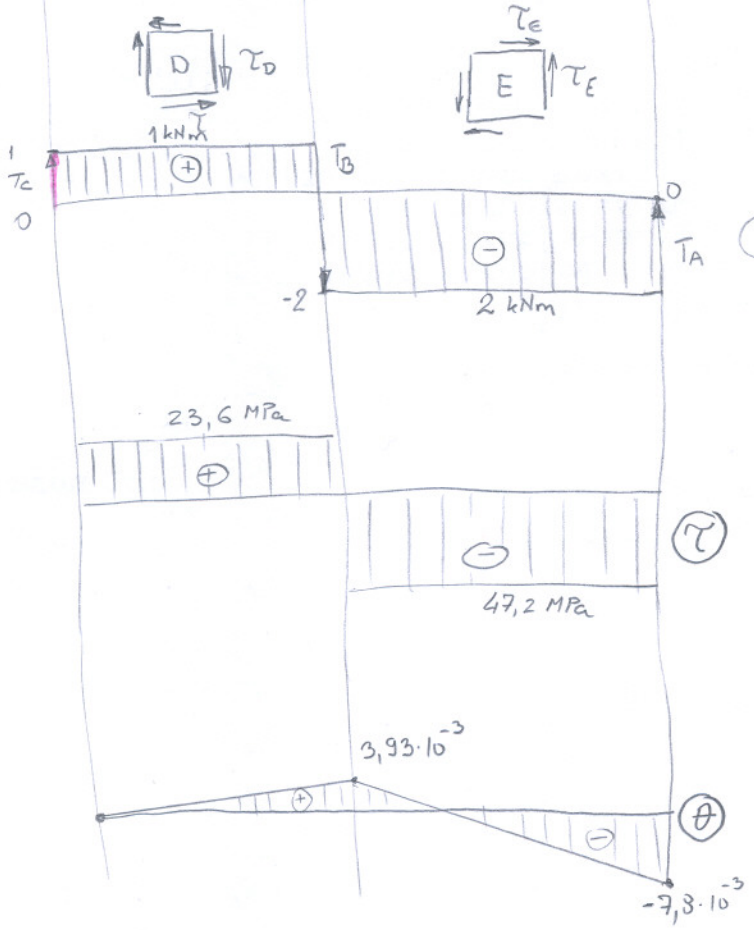


Statički uslov ravnoteže

$$\Sigma T_z = 0 \quad -T_C + T_B - T_A = 0$$

$$T_C = T_B - T_A$$

$$= 3 - 2 = 1\ kNm$$



Maksim. tangencijalni napon

$$\tau_{CD} = \frac{T_{CB}}{W_0} = \frac{100\,kNcm}{42,41\,cm^3} = 2,36\ \frac{kN}{cm^2}$$

$$W_0 = \frac{d_3^3\,\pi}{16} = \frac{(6cm)^3\,\pi}{16} = 42,41\,cm^3$$

$$\tau_{BA} = \frac{T_{BA}}{W_0} = \frac{200\,kNcm}{42,41cm^3} = 4,72\ \frac{kN}{cm^2}$$

Taug. naponi u elementima D i E:

$$\tau_D = \frac{T_{CB}}{I_0}\cdot r_D = \frac{100\,kNcm}{127,23\,cm^4}\cdot 3cm$$

$$I_0 = \frac{d^4\,\pi}{32} = \frac{(6cm)^4\,\pi}{32} = 127,23\,cm^4$$

$$\tau_D = 2,36\ \frac{kN}{cm^2} = 23,6\ MPa$$

$$\tau_E = \frac{T_{BA}}{I_0}\cdot r_E = \frac{200\,kNcm}{127,23\,cm^4}\cdot 2cm$$

$$\tau_E = 3,14\ \frac{kN}{cm^2}$$

$$\tau_E = 31,4\ MPa$$

Ugao uvijanja kraja A u odnosu na C

$$\theta_{B/C} = \frac{T_{CB}\cdot 40cm}{G\cdot I_0} =$$

$$= \frac{100\,kNcm\cdot 40cm}{8\cdot10^3\,\frac{kN}{cm^2}\cdot 127,23cm^4},$$

$$= 3,93\cdot10^{-3}$$

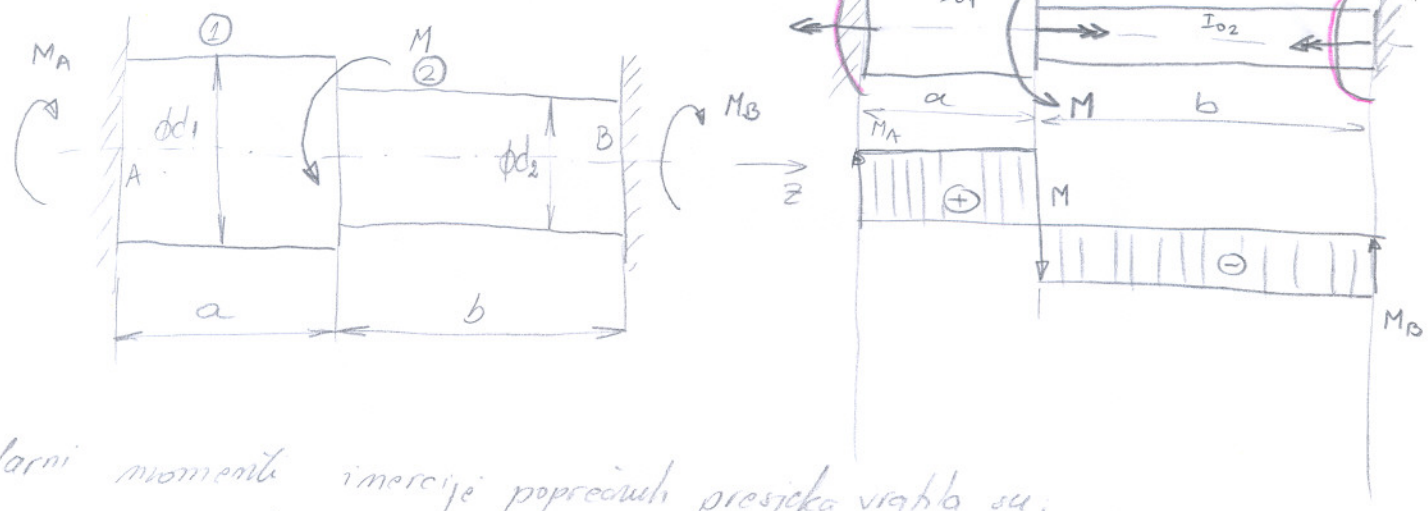
Ugao uvijanja kraja A prema B

$$\theta_{A/B} = \frac{-200\,kNcm\,60cm}{8\,10^3\,\frac{kN}{cm^2}\cdot 127,23cm^4}$$

$$\theta_{A/B} = -11,79\cdot10^{-3}$$

Ukupan ugao uvijanja: $\theta_{A/C} = \theta_{B/C} + \theta_{A/B} = 3,93\cdot10^{-3} - 11,79\cdot10^{-3} = -7,86\cdot10^{-3}\ [rad]$

Odrediti max obrtni moment M vratila promjenjivog kružnog poprečnog presjeka $d_1 = 60\,mm$ $d_2 = 40\,mm$, uklještenog na oba kraja, ako je poznato $a = 120\,mm$, $\tau_d = 60\,MPa$, $b = 200\,mm$



Polarni momenti inercije poprečnih presjeka vratila su:

$$I_{o2} = \frac{d_2^4 \pi}{32} = \frac{40^4 \pi}{32} = 251 \cdot 10^3\ mm^4$$

$$I_{o1} = \frac{d_1^4 \pi}{32} = \frac{60^4 \pi}{32} = 1272 \cdot 10^3\ mm^4$$

Iz uslova ravnoteže $\Sigma M_z = 0$

$$-M_A + M - M_B = 0 \quad (1) \Rightarrow M_B = M - M_A$$

Dodatni uslov: $\theta_{B/A} = 0$; (2)

$$\theta_{B/A} \quad \frac{M_A \cdot a}{G \cdot I_{o1}} - \frac{M_B \cdot b}{G \cdot I_{o2}} = \frac{M_A \cdot a}{G \cdot I_{o1}} - \frac{M \cdot b}{G \cdot I_{o1}} + \frac{M_A \cdot b}{G \cdot I_{o2}} = 0$$

Iz (2) $$M_A \left(\frac{a}{I_{o1}} + \frac{b}{I_{o2}} \right) = \frac{M \cdot b}{I_{o2}}$$

$$M_A = \frac{M \cdot b}{I_{o2} \left(\frac{a}{I_{o1}} + \frac{b}{I_{o2}} \right)} = \frac{M \cdot b \, I_{o1}}{I_{o2} \cdot a + b \cdot I_{o1}}$$

$$M_B = \frac{M \cdot a \cdot I_{o2}}{I_{o1} \cdot b + a \cdot I_{o2}}$$

$$\tau_{①} = \frac{M_A}{W_{o1}} = \frac{M_A}{I_{o1}} \cdot \frac{d_1}{z} = \frac{M \cdot b}{I_{o2} \cdot a + b \cdot I_{o1}} \cdot \frac{d_1}{z} \leq \tau_{dozv}$$

$$= \frac{M \cdot 20cm \cdot 20cm}{25,1\,cm^4 \cdot 12cm + 20cm \cdot 127,2\,cm^4} \leq 6\,\frac{kN}{cm^2}$$

$$\frac{M \cdot 20cm \cdot 3cm}{2\,845,2\,cm^5} \leq 6\,\frac{kN}{cm^2}$$

$$M_I \leq 284,52\,kNcm$$

$$\tau_{②} = \frac{M_B}{W_{o2}} = \frac{M_B}{I_{o2}} \cdot \frac{d_2}{z} = \frac{M \cdot a \cdot 3cm}{I_{o2} \cdot a + b \cdot I_{o1}} \cdot \frac{d_2}{z} \leq \tau_{dozv}$$

$$\frac{M \cdot 12cm \cdot 2cm}{2\,845,2\,cm^5} \leq 6\,\frac{kN}{cm^2}$$

$$M_{II} \leq 711,25\,kNcm$$

Usvaja se manja vrijednost momenta $M = 284,52\,kNcm$

3.

$$\tau = \frac{T}{I_0} \cdot r$$

$$P = T \cdot \omega$$

$$P = 300\ kW = 300\,000\ \frac{Nm}{s}$$

$$n = 400\ \frac{obr}{min} = 400 \cdot \frac{2\tilde{\pi}\ rad}{60\ s} = 41{,}88\ \frac{rad}{s}$$

$$T = 7\,161{,}97\ Nm$$

$$I_0 = \frac{\tilde{\pi} d^4}{32}$$

$$W_0 = \frac{I_0}{r} = \frac{\tilde{\pi} d^3}{16}$$

$$\tau \cdot W_0 = T$$

$$W_0 = \frac{T}{\tau}$$

$$\frac{\tilde{\pi} d^3}{16} = \frac{T}{\tau}$$

$$d = \sqrt[3]{\frac{T \cdot 16}{\tau \cdot \tilde{\pi}}}$$

$$d = 80{,}5\ mm$$

Vratilo motora dizajnirano je da prenosi snagu od 300kW pri broju obrtaja u minuti: $n=400min^{-1}$. Ako je maksimalni tangencijalni napon ograničen na 70 MPa odrediti minimalan prečnik punog vratila.