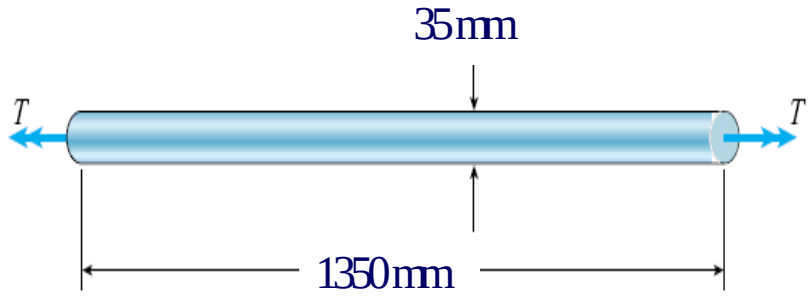
Primjer 4.1

Puni štap kružnog poprečnog presjeka, prečnika 40 mm, dužine 1350 mm i modula klizanja 80 GPa, opterećen je momentom uvijanja na svojim krajevima, kao što je dato na slici. Odrediti:
a) Maksimalan tangencijalni napon u šipki, te ugao uvijanja ako je moment uvijanja 340 Nm
b) Maksimalan mogući moment uvijanja, ako je dozvoljeni tangencijalni napon 4 MPa, a maksimalni dozvoljeni ugao uvijanja 2.5 stepeni



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$L := 1350mm$ $T := 340N \cdot m$

$D := 40mm$ $\tau_{doz} := 40MPa$

$G := 80GPa$ $\varphi_{doz} := 2.5°$

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a)

$$\tau_{max} = \frac{T}{W_o} = \frac{16T}{D^3\pi}$$

$$\tau_{max} := \frac{16T}{D^3\pi} = 27.056\,MPa$$

$$\varphi = \frac{T \cdot L}{G \cdot I_o} \qquad I_o = \frac{D^4 \pi}{32}$$

$$\varphi := \frac{32T \cdot L}{G \cdot D^4 \pi} = 1.308\,°$$

b)

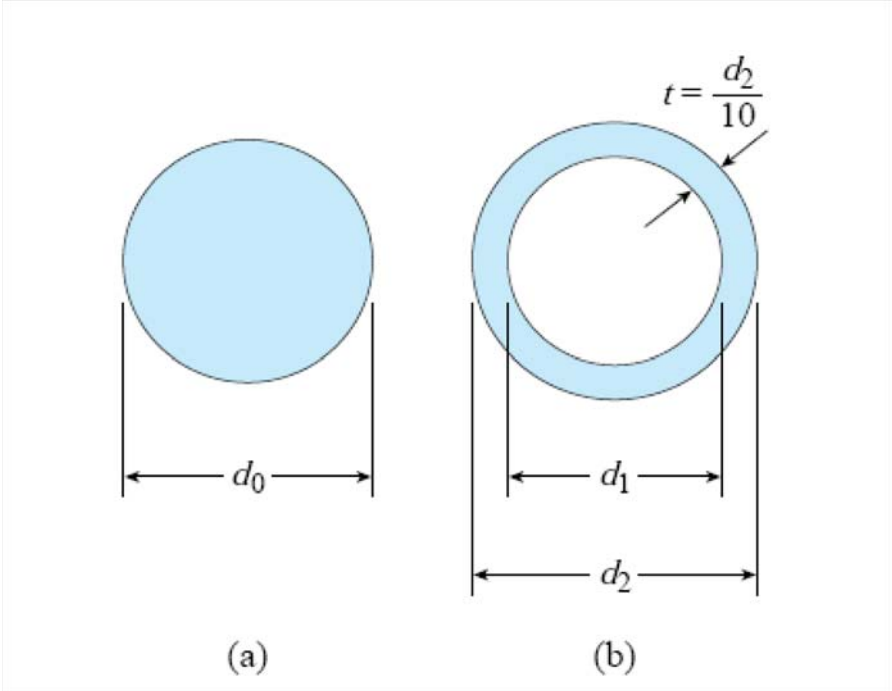
$$T_{max1} := \frac{\tau_{doz} \cdot D^3 \cdot \pi}{16} = 502.655\,N \cdot m$$

$$T_{max2} := \frac{\varphi_{doz} \cdot G \cdot D^4 \pi}{32L} = 649.85\,N \cdot m$$

$$T_{max} := \min\left(T_{max1}, T_{max2}\right) = 502.655\,N \cdot m$$

Primjer 4.2

Vratilo cilindričnog poprečnog presjeka od čelika, izrađeno u dvije varijante, kao puno i šuplje (slika), treba prenese moment uvijanja od 1200 Nm bez prekoračenja dozvoljenog tangencijalnog napona od 40MPa i dozvoljenog uzdužnog uvijanja od 0.75/m.
Treba odrediti:
a) Prečnik punog vratila
b) Potrebni vanjski prečnik šupljeg vratila ako je debljina stjenke vratila jedna desetina vanjskog prečnika
c) Odnos prečnika (*d*2/*d*1) i težina oba vratila



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$T := 1200N \cdot m$ $E := 200GPa$ $\nu := 0.3$

$\tau_{doz} := 40MPa$

$G := \frac{E}{2 \cdot (1 + \nu)} = 76.923\,GPa$

$\theta_{doz} := 0.75 \frac{^\circ}{m}$

****

a) puno vratilo

$$\tau_{doz} = \frac{T}{W_o} = \frac{16T}{d_0^{\,3}\pi}$$

$$d_{01} := \sqrt[3]{\frac{16T}{\tau_{doz}\cdot\pi}} = 53.46\,mm$$

$$\theta_{doz} = \frac{T}{G\cdot I_o} \qquad I_o = \frac{d_0^{\,4}\pi}{32}$$

$$d_{02} := \sqrt[4]{\frac{32T}{G\cdot\pi\cdot\theta_{doz}}} = 59.026\,mm$$

$$d_0 := \max\left(d_{01}, d_{02}\right) = 59.026\,mm$$

b) šuplje vratilo

$$d_1 = d_2 - 2t = 0.8d_2$$

$$I_0 = \left(d_2^{\,4} - d_1^{\,4}\right)\cdot\frac{\pi}{32} = \left(1 - 0.8^4\right)\cdot\frac{\pi}{32}\,d_2^{\,4}$$

$$\tau_{doz} = \frac{T\cdot\frac{d_2}{2}}{I_0} = \frac{T}{\left(1 - 0.8^4\right)\cdot\frac{\pi}{16}\cdot d_2^{\,3}}$$

$$d_{21} := \sqrt[3]{\frac{16T}{\tau_{doz}\cdot\pi\cdot\left(1-0.8^4\right)}} = 63.726\,mm$$

$$\theta_{doz} = \frac{T}{G\cdot I_o}$$

$$d_{22} := \sqrt[4]{\frac{32T}{G\cdot\pi\cdot\left(1-0.8^4\right)\cdot\theta_{doz}}} = 67.338\,mm$$

$$d_2 := \max\left(d_{21}, d_{22}\right) = 67.338\,mm$$

$$d_1 := 0.8 d_2 = 53.87\,mm$$

c) Odnos prečnika i težina

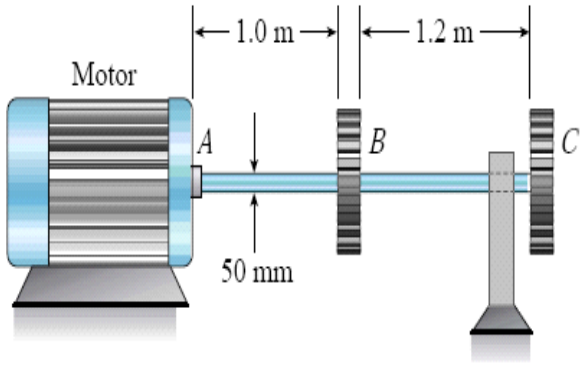
$$\frac{d_2}{d_0} = 1.141$$

$$\frac{W_{šuplje}}{W_{puno}} = \frac{\gamma\cdot V_{šuplje}}{\gamma\cdot V_{puno}} = \frac{A_{šuplje}}{A_{puno}} = \frac{\left(d_2^{\,2} - d_1^{\,2}\right)\cdot\frac{\pi}{4}}{d_0^{\,2}\cdot\frac{\pi}{4}}$$

$$\frac{\left(d_2^{\,2} - d_1^{\,2}\right)}{d_0^{\,2}} = 0.469$$

Primjer 4.3

Puno vratilo *ABC* prečnika 50 mm se pokreće motorom snage 50 kW i frekvencije 10 Hz u tački *A*. Zupčanici *B* i *C* pokreću uređaje koji potražuju snagu od 35 i 15 kW, respektivno. Odrediti najveći tangencijalni napon τmax u vratilu i ugao uvijanja ϕ_{AC} između motora *A* i zupčanika *C*. Uzeti da je G=80 Gpa.



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$P := 50kW$

$P_B := 35kW$

$P_C := 15kW$

$L_{AB} := 1000mm$

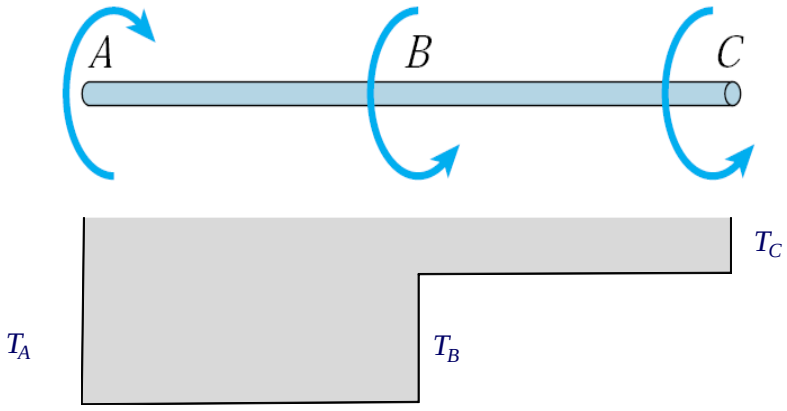
$L_{BC} := 1200mm$

$G := 80GPa$

$f := 10Hz$

$d := 50mm$

****



$$T_A := \frac{P}{2\pi \cdot f} = 795.775 \cdot N \cdot m$$

$$T_B := \frac{P_B}{2\pi \cdot f} = 557.042 \cdot N \cdot m$$

$$T_C := \frac{P_C}{2\pi \cdot f} = 238.732 \cdot N \cdot m$$

$$T_{AB} := T_A = 795.775 \cdot N \cdot m$$

$$T_{BC} := T_C = 238.732 \cdot N \cdot m$$

$$I_0 := \frac{d^4 \cdot \pi}{32}$$

$$\tau_{AB} := \frac{T_{AB}}{\left(\frac{d^3 \pi}{16}\right)} = 32.423 \cdot MPa$$

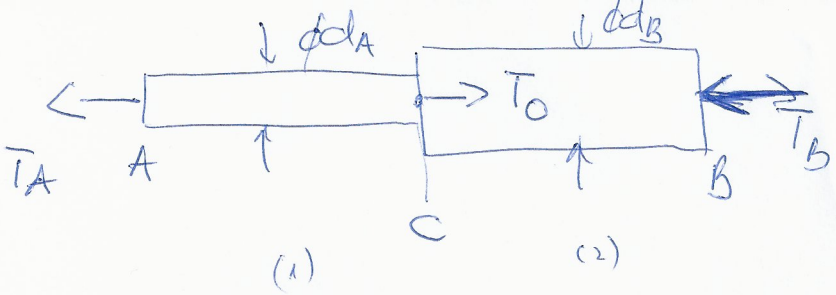
$$\phi_{AB} := \frac{T_{AB} \cdot L_{AB}}{G \cdot I_0} = 0.016$$

$$\tau_{BC} := \frac{T_{BC}}{\left(\frac{d^3 \pi}{16}\right)} = 9.727 \cdot MPa$$

$$\phi_{BC} := \frac{T_{BC} \cdot L_{BC}}{G \cdot I_0} = 5.836 \times 10^{-3}$$

$$\tau_{max} := \max\left(\tau_{AB}, \tau_{BC}\right) = 32.423 \cdot MPa$$

$$\phi_{AC} := \phi_{AB} + \phi_{BC} = 0.022 \qquad \phi_{AC} = 1.263 \cdot °$$



Jednačine ravnoteže

$$\bar{T}_A + \bar{T}_B = \bar{T}_0 \qquad (1)$$

Jednačine kompatibilnosti

$$\varphi_1 + \varphi_2 = 0 \qquad (2)$$

ili

superpozicija

$$\varphi_{T_0} = \varphi_{T_B} \qquad (3)$$

$$\varphi_{T_0} = \frac{T_0 L_{AC}}{G I_{OAC}} \qquad \varphi_{T_B} = -\frac{T_B \cdot L_{AC}}{G I_{OAC}} - \frac{T_B \cdot L_{BC}}{G I_{OBC}} \qquad (4)$$

$$(4) \rightarrow (3) \Rightarrow \quad T_B = T_0 \frac{L_{AC} I_{OBC}}{L_{AC} I_{OBC} + L_{BC} I_{OAC}} \qquad (5)$$

$$(5) \Rightarrow (1) \Rightarrow \quad T_A = T_0 \frac{L_{BC} \cdot I_{OAC}}{L_{AC} I_{OBC} + L_{BC} I_{OAC}} \qquad (6)$$

$$\tau_{AC} = \frac{T_A d_A}{2 I_{OAC}} \qquad \tau_{BC} = \frac{T_B d_B}{2 I_{OBC}} \qquad (7)$$

(5); (6) $\Rightarrow$ (7)

$$\tau_{AC} = \frac{T_0 L_{BC} d_A}{2(L_{AC} I_{OBC} + L_{BC} I_{OAC})}$$

$$\tau_{BC} = \frac{T_0 L_{AC} d_B}{2(L_{AC} I_{OBC} + L_{BC} I_{OAC})}$$

$$\max \tau = \max(\tau_{AC}, \tau_{BC}) = \max(L_{BC} \cdot d_A, L_{AC} d_B)$$

$$\gamma_C = \frac{T_A \cdot L_{AC}}{G \cdot I_{OAC}} = \frac{T_B \cdot L_{BC}}{G \cdot I_{OBC}} = T_0 \frac{L_{AC} L_{BC}}{G(L_{AC} I_{OBC} + L_{BC} I_{OAC})}$$

Primjer 4.5

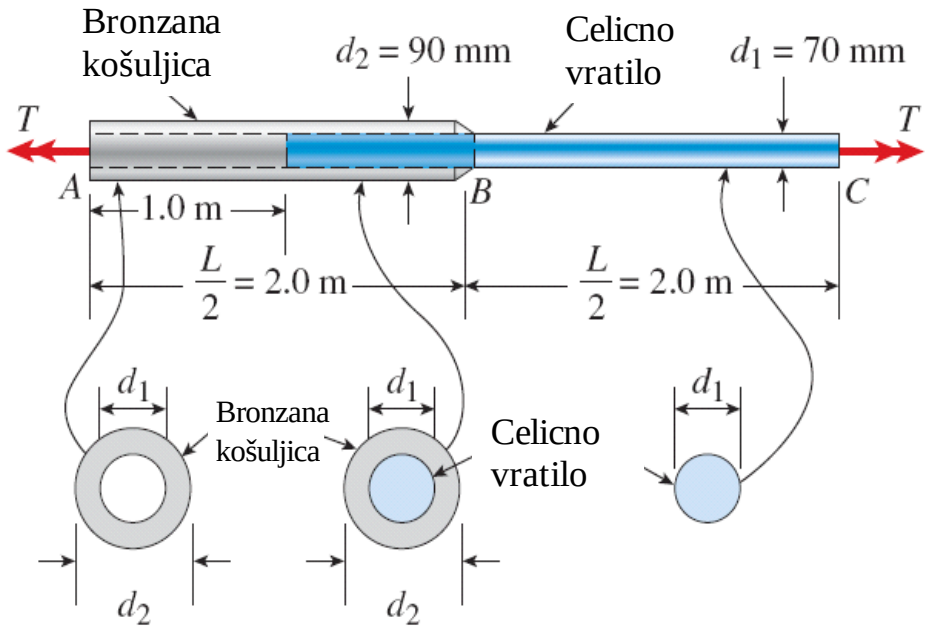
Vratilo izrađeno od čelika ($G_č$=80 GPa), dužine 3 m, je trećinom dužine uvučeno u bronzanu košuljicu (G_b=40 GPa) koja je čvrsto povezana s vratilom. Vanjski prečnici vratila i košuljice su d_1=70 mm i d_2=90 mm, respektivno. Odrediti:

a) Dozvoljeni moment uvijanja T_1 koji se može primijeniti na krajeve ako je dozvoljeni napon uvijanja između krajeva 8 stepeni.

b) Dozvoljeni moment uvijanja T_2 ako je dozvoljni tangencijalni napon bronze τ_b=70 Mpa

c) Dozvoljeni moment uvijanja T_3 ako je dozvoljni tangencijalni napon čelika $\tau_č$=110 Mpa

d) Dozvoljeni moment uvijanja *T*max ako moraju biti zadovoljena sva tri uslova pod a), b), c)



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$L_1 := 1m$

$L_2 := 1m$

$L_3 := 2m$

$d_1 := 70mm$

$d_2 := 90mm$

$G_č := 80GPa$

$G_b := 40GPa$

$\phi_{doza} := 8°$

$\tau_{dozb} := 70MPa$

$\tau_{dozc} := 110MPa$

****

a)

$$I_{0č} := \frac{{d_1}^4 \cdot \pi}{32} = 2.357 \times 10^6 mm^4$$

$$I_{0b} := \frac{\left({d_2}^4 - {d_1}^4\right) \cdot \pi}{32} = 4.084 \times 10^6 mm^4$$

$$\phi_a = \frac{T_1 \cdot L_1}{G_b \cdot I_{0b}} + \frac{T_č \cdot L_2}{G_č \cdot I_{0č}} + \frac{T_1 \cdot L_3}{G_č \cdot I_{0č}}$$

$$T_č = T_1 \cdot \frac{G_č \cdot I_{0č}}{G_č \cdot I_{0č} + G_b \cdot I_{0b2}}$$

$$\phi_a = \frac{T_1 \cdot L_1}{G_b \cdot I_{0b}} + \frac{L_2}{G_č \cdot I_{0č}} \cdot \left(T_1 \cdot \frac{G_č \cdot I_{0č}}{G_č \cdot I_{0č} + G_b \cdot I_{0b}} \right) + \frac{T_1 \cdot L_3}{G_č \cdot I_{0č}}$$

$$T_1 := \frac{\phi_{doza}}{\frac{L_2}{G_b \cdot I_{0b} + G_č \cdot I_{0č}} + \frac{L_3}{G_č \cdot I_{0č}} + \frac{L_1}{G_b \cdot I_{0b}}} = 7.135 \times 10^3 N \cdot m$$

b)

$$\tau_{dozb} = \frac{T_2 \cdot \frac{d_2}{2}}{I_{0b}} \qquad T_{21} := \frac{2\tau_{dozb} \cdot I_{0b}}{d_2} = 6.353\,kN \cdot m$$

$$\tau_{dozb} = \frac{T_b \cdot \frac{d_2}{2}}{I_{0b}}$$

$$\tau_1 = T_2 \frac{G_b \cdot I_{0b}}{G_č \cdot I_{0č} + G_b \cdot I_{0b}} \cdot \frac{d_2}{2 \cdot I_{0b}}$$

$$T_b = T_2 \cdot \frac{G_b \cdot I_{0b}}{G_č \cdot I_{0č} + G_b \cdot I_{0b}}$$

$$T_{22} := \frac{2 \cdot \tau_{dozb} \cdot \left(G_b \cdot I_{0b} + G_č \cdot I_{0č}\right)}{G_b \cdot d_2} = 13.686\,kN \cdot m$$

$$T_2 := \min\left(T_{21}, T_{22}\right) = 6.353\,kN \cdot m$$

c)

$$\tau_{dozc} = \frac{T_č \cdot \frac{d_2}{2}}{I_{0b}}$$

$$\tau_{dozc} = T_3 \frac{G_č \cdot I_{0č}}{G_č \cdot I_{0č} + G_b \cdot I_{0b}} \cdot \frac{d_1}{2 \cdot I_{0č}}$$

$$T_č = T_3 \cdot \frac{G_č \cdot I_{0č}}{G_č \cdot I_{0č} + G_b \cdot I_{0b}}$$

$$T_{31} := \frac{2 \cdot \tau_{dozc} \cdot \left(G_b \cdot I_{0b} + G_č \cdot I_{0č}\right)}{d_1 \cdot G_č} = 13.826\,kN \cdot m$$

$$\tau_{dozc} = \frac{T_3 \cdot \frac{d_1}{2}}{I_{0č}} \qquad T_{32} := \frac{2\tau_{dozc} \cdot I_{0č}}{d_1} = 7.408\,kN \cdot m$$

$$T_3 := \min\left(T_{31}, T_{32}\right) = 7.408\,kN \cdot m$$

d)

$$T_{max} := \min\left(T_1, T_2, T_3\right) = 6.353\,kN \cdot m$$