

Str.61 Naloge 13 a - č

Reši enačbe:

- 3 a) $\log x + \log(x+3) = \log(x-1) + \log(x+2)$
b) $\log(2x+3) + \log(3x-1) = \log x + \log(6x+4)$
c) $\log(x+3) + \log(x+2) = \log(x+6) + \log(x-1)$
č) $\log 2 + 2 \log(x+1) = \log(2x^2 + 4x + 2)$

Rešiti moramo enačbe, prikaterih velja:

- (1) $a^y = x \Leftrightarrow y = \log_a x, \quad x > 0, \quad a > 0, \quad a \neq 1$
(2) $\log_a 1 = 0$
(3) $\log_a a = 1$
(4) $\log_a(x+y) = \log_a x + \log_a y$
(5) $\log_a x^n = n \log_a x$
(6) $\log_a \left(\frac{x}{y} \right) = \log_a x - \log_a y$
(7) $\log_a x = \frac{\log_b x}{\log_b a}$
(8) $\log_{10} x = \log x$

Rešitev

13 a) $\log x + \log(x+3) = \log(x-1) + \log(x+2)$

Po pravilu (4) dobimo:

$$\log(x \cdot (x+3)) = \log((x-1)(x+2)) / \text{antilogaritmiramo}$$

$$x(x+3) = (x-1)(x+2)$$

$$x^2 + 3x = x^2 + x - 2$$

$$2x = -2$$

$x = -1$ Rešitev ne ustreza, ker $\log(-1)$ ni definiran.

b) $\log(2x+3) + \log(3x-1) = \log x + \log(6x+4)$

Po (4) dobimo:

$$\log((2x+3)(3x-1)) = \log(x(6x+4))$$

$$(2x+3)(3x-1) = x(6x+4)$$

$$6x^2 + 7x - 3 = 6x^2 + 4$$

$$7x = 7$$

$x = 1$ ustreza

c) $\log(x+3) + \log(x+2) = \log(x+6) + \log(x-1)$

Po (4):

$$\log((x+3)(x+2)) = \log((x+6)(x-1)) / anti log aritmirmamo$$

$$(x+3)(x+2) = (x+6)(x-1)$$

$$x^2 + 5x + 6 = x^2 + 5x - 6$$

$$6 = -6$$

$12 \neq 0$ Enačba nima rešitve

č) $\log 2 + 2 \log(x+1) = \log(2x^2 + 4x + 2)$

Po (5):

$$\log 2 + \log(x+1)^2 = \log(2x^2 + 4x + 2)$$

Po (4):

$$\log(2 \cdot (x+1)^2) = \log(2x^2 + 4x + 2) / aniti log.$$

$$2(x^2 + 2x + 1) = 2x^2 + 4x + 2$$

$$2x^2 + 4x + 2 = 2x^2 + 4x + 2$$

$$0 = 0$$

Rešitev so vsi x -i za katere velja, da so vsi argumenti pozitivni:

$$(x+1 > 0) \wedge (2x^2 + 4x + 2 > 0)$$

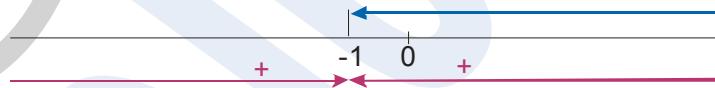
$$x > -1 \wedge D = 16 - 16 = 0$$

$$x_{1,2} = \frac{-4 \pm 0}{2 \cdot 2}$$

$$x_{1,2} = -1$$

$$x > -1 \wedge (x+1)^2 > 0$$

$$x > -1 \wedge ((x > -1) \wedge (x < -1))$$



Skupni pogoj je $x > 0$, zato so rešitev vsi $x > -1$ ali $x \in (-1, \infty)$