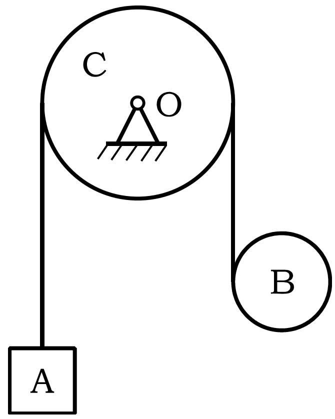
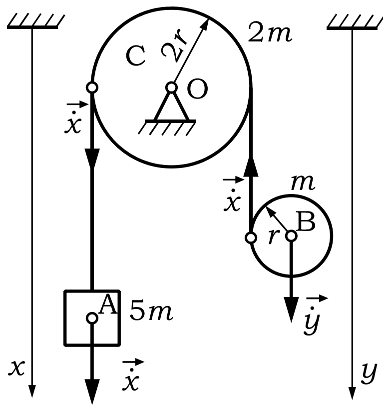
Oko diska C poluprečnika 2r, mase 2m koji se može obrtati oko horizontalne nepokretne ose koja prolazio kroz tačku O omotano je uže na čijem jednom kraju visi teret A, mase 5m, a drugi kraj je omotan oko diska B, poluprečnika r, mase m. Sistem se kreće u vertikalnoj ravni pod dejstvom zemljine teže. Odrediti ubrzanje tereta A. Težinu užeta i trenje zanemariti.





$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{x}}\right)+\frac{\partial E_K}{\partial x}=Q_x$$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{y}}\right)+\frac{\partial E_K}{\partial y}=Q_y$$

$$E_{KA}=\frac{5m\dot{x}^2}{2}$$

$$E_{KC}=\frac{1}{2}\left(\frac{2m(2r)^2}{2}\right)\left(\frac{\dot{x}}{2r}\right)^2=\frac{m\dot{x}^2}{2}$$

$$E_{KB}=\frac{m\dot{y}^2}{2}+\frac{1}{2}\left(\frac{mr^2}{2}\right)\left(\frac{\dot{x}+\dot{y}}{r}\right)^2=$$

$$=\frac{m\dot{x}^2}{4}+\frac{m\dot{x}\dot{y}}{2}+\frac{3m\dot{y}^2}{4}$$

$$E_K=\frac{13m\dot{x}^2}{4}+\frac{m\dot{x}\dot{y}}{2}+\frac{3m\dot{y}^2}{4}$$

$$Q_x=\frac{\delta A_x}{\delta x}=5mg\ ;\ Q_x=\frac{\delta A_x}{\delta x}=mg$$

$$\frac{13}{2}m\ddot{x}+\frac{1}{2}m\ddot{y}=5mg\,\Big|\cdot\frac{3}{m}$$

$$\frac{1}{2}m\ddot{x}+\frac{3}{2}m\ddot{y}=mg\,\Big|\cdot-\frac{1}{m}$$

$$\frac{39}{2}\ddot{x}+\frac{3}{2}\ddot{y}=15g$$

$$-\frac{1}{2}\ddot{x}-\frac{3}{2}\ddot{y}=-g$$

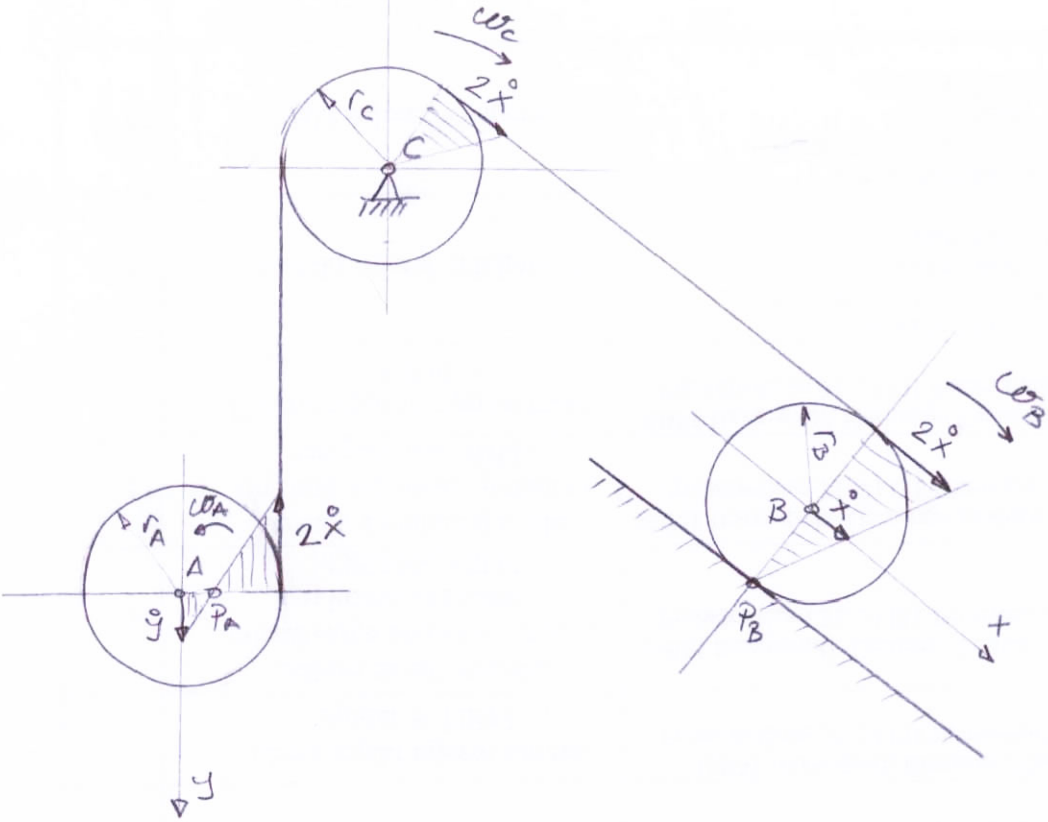
$$\frac{38}{2}\ddot{x}=14g$$

$$\ddot{x}=\frac{14}{19}g$$

Neistegljivo uže zanemarljive mase, omotano je oko homogenih cilindara A i B i prebačeno preko kotura C, kako je prikazano na slici. Cilindar B se može kotrljati bez klizanja po strmoj ravni nagiba α. Sistem se kreće pod dejstvom sile teže.

Dato je: $r_A = r_B = r_C = r$, $m_A = m$, $m_B = \frac{m}{2}$, $m_C = \frac{m}{4}$.

Zanemarivši trenje u ležaju kotura C odrediti ubrzanje cilindara A i B.



L. j. II vrste:

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{x}}\right) + \frac{\partial E_K}{\partial x} = Q_x$$

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{y}}\right) + \frac{\partial E_K}{\partial y} = Q_y$$

Kin. veze:

$$\omega_B = \frac{2\dot{x}}{r} \qquad \omega_A = \frac{2\dot{x} + \dot{y}}{r} \qquad \omega_C = \frac{2\dot{x}}{r}$$

$E_k = E_{kA} + E_{kB} + E_{kC}$

$$E_{kA} = \frac{m_A v_{AC}^2}{2} + \frac{J_A \cdot \omega_A^2}{2} = \frac{m\dot{y}}{2} + \frac{1}{4} m(2\dot{x}+\dot{y})^2 = m(\dot{x}^2 + \dot{x}\dot{y} + \frac{3}{4}\dot{y}^2)$$

$$E_{kB} = \frac{m_B \cdot v_{BC}^2}{2} + \frac{J_B \cdot \omega_B^2}{2} = \frac{m\dot{x}^2}{4} + \frac{1}{8} m\dot{x}^2 = \frac{3}{8} m\dot{x}^2$$

$$E_{kC} = \frac{1}{2} \frac{m_C r_C^2}{2} \cdot \omega_C^2 = \frac{m}{4}\dot{x}^2$$

$$E_k = \frac{13}{8} m\dot{x}^2 + m\dot{x}\dot{y} + \frac{3}{4} m\dot{y}^2$$

$$\left(\frac{\partial E_k}{\partial \dot{x}}\right) = \frac{13}{4} m\dot{x} + m\dot{y} \qquad \frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}}\right) = \frac{13}{4} m\ddot{x} + m\ddot{y}$$

$$\left(\frac{\partial E_k}{\partial \dot{y}}\right) = \frac{3}{2} m\dot{y} + m\dot{x} \qquad \frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{y}}\right) = \frac{3}{2} m\ddot{y} + m\ddot{x}$$

Generalisane sile

$$Q_x = \frac{\delta A_x}{\delta x}$$

$$\delta A_x = G_B \cdot \sin\alpha \cdot \delta x = \frac{m}{2} g \cdot \sin\alpha \cdot \delta x$$

$$Q_x = \frac{1}{2} m \cdot g \cdot \sin\alpha$$

$$Q_y = \frac{\delta A_y}{\delta y}$$

$$\delta A_y = G_A \cdot \delta y = m \cdot g \cdot \delta y$$

$$Q_y = m \cdot g$$

L.J.:

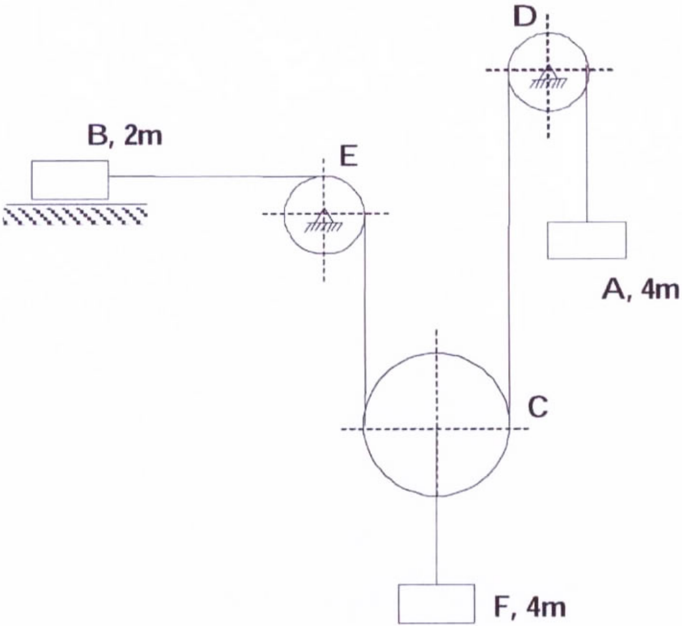
$$\frac{13}{4} m\ddot{x} + m\ddot{y} = \frac{1}{2} m \cdot g \cdot \sin\alpha$$

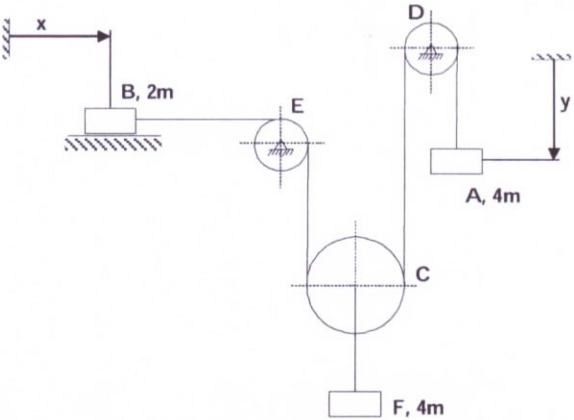
$$m\ddot{x} + \frac{3}{2} m\ddot{y} = mg$$

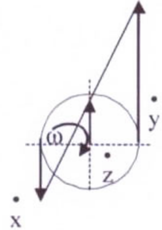
Rješavanjem jedn. se dobije:

$$\ddot{y} = g\left(\frac{26}{31} - \frac{4}{31}\sin\alpha\right) \qquad \ddot{x} = -g\left(\frac{8 - 6\cdot\sin\alpha}{31}\right)$$

Sistem prikazan na slici sastoji se od lakih koturova C, D i E, tereta A, mase 4m, tereta B, mase 2m i tereta F, mase 4m. Teret B klizi po horizontalnoj ravni koeficijenta trenja μ, vezan je lakim užetom koje je prebačeno preko koturova i svojim drugim krajem vezano za teret A. Teret F je vezan lakim užetom za centar kotura C. Odrediti ubrzanja tereta.







$$\omega = \frac{\dot{x}}{p} = \frac{\dot{y}}{2R - p} = \frac{\dot{x} + \dot{y}}{2R} = \frac{\dot{y} - \dot{x}}{2(R-p)} = \frac{\dot{z}}{R-p}$$

$$\omega = \frac{\dot{x} + \dot{y}}{2R} \qquad \dot{\omega} = \frac{\ddot{x} + \ddot{y}}{2R}$$

$$\dot{z} = \frac{\dot{y} - \dot{x}}{2} \qquad \ddot{z} = \frac{\ddot{y} - \ddot{x}}{2} \qquad \partial z = \frac{\partial y - \partial x}{2}$$

$$E_k = E_{kA} + E_{kB} + E_{kF} \qquad E_k = \frac{1}{2} 4m \dot{y}^2 + \frac{1}{2} 2m \dot{x}^2 + \frac{1}{2} 4m \left(\frac{\dot{y} - \dot{x}}{2} \right)^2$$

$$\underline{\underline{E_k = \frac{3}{2} m \dot{x}^2 + \frac{5}{2} m \dot{y}^2 - m \dot{x} \dot{y}}}$$

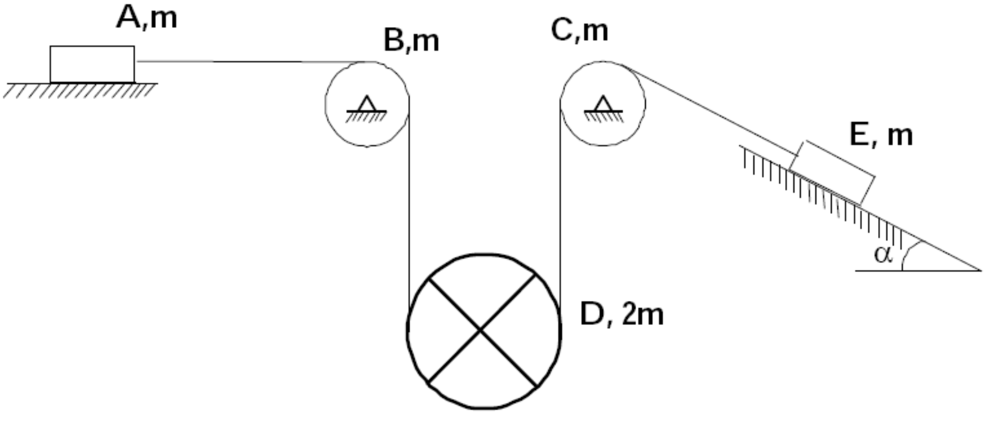
$$\partial A = \partial A_A + \partial A_B + \partial A_F \qquad \partial A = -2\mu m g \partial x - 4mg \frac{\partial y - \partial x}{2} + 4mg \partial y$$

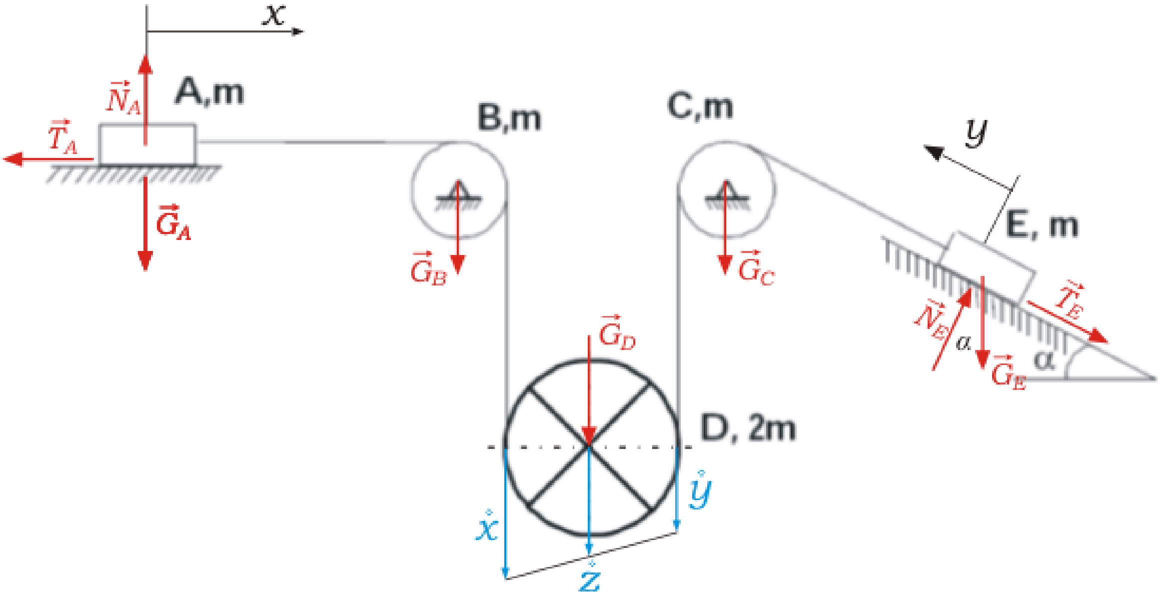
$$\underline{\underline{\partial A = 2mg(1-\mu)\partial x + 2mg \partial y}}$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}} \right) - \frac{\partial E_k}{\partial x} = Q_x \qquad 3\ddot{x} - \ddot{y} = 2g(1-\mu) \qquad \ddot{x} = \frac{(6 - 5\mu)}{7} g$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{y}} \right) - \frac{\partial E_k}{\partial y} = Q_y \qquad \underline{\underline{-\ddot{x} + 5\ddot{y} = 2g}} \qquad \underline{\underline{\ddot{y} = \frac{(4 - \mu)}{7} g}}$$

Sistem prikazan na slici sastoji se od masa *A* i *E*, koturova *B* i *C*, poluprečnika *r*, i točka *D*, poluprečnika *R*. Točak *D* je ukupne mase $2m$, poluprečnika *R*, sa stoji se od obruča, mase $m_O = \frac{m}{2}$, i dva homogena štapa masa po $m_t = \frac{3}{4}m$. Koturove *B* i *C* smatrati homogenim, masa po *m*. Teret *A*, mase *m*, klizi po horizontalnoj hrapavoj ravni koeficijenta trenja $\mu_1 = \frac{1}{12}$, a teret *E*, mase *m*, klizi po hrapavoj strmoj ravni koeficijenta trenja $\mu_2 = \frac{\sqrt{3}}{12}$ i nagibnog ugla $\alpha = 30°$. Veze među elementima ostvarene su pomoću lake nerastegljive užadi. Odrediti ubrzanja tereta *A* i *E*.





Momenti inercije tijela u sistemu:

$$I_B = \frac{mr^2}{2} \qquad I_C = \frac{mr^2}{2} \qquad I_D = \frac{mR^2}{2} + 2\left[\frac{1}{12}\frac{3m}{4}\;2R^{\;2}\right] = mR^2$$

Kinematske veze:

$$\dot{z} = \frac{\dot{x}+\dot{y}}{2} \qquad \omega_D = \frac{\dot{x}-\dot{y}}{2R} \qquad \omega_B = \frac{\dot{x}}{r} \qquad \omega_C = \frac{\dot{y}}{r}$$

Kinetička energija sistema:

$$E_K = E_{KA} + E_{KB} + E_{KC} + E_{KD} + E_{KE} = \frac{m\dot{x}^2}{2} + \frac{I_B\omega_B^2}{2} + \left[\frac{I_D\omega_D^2}{2} + \frac{2m\dot{z}^2}{2}\right] + \frac{I_C\omega_C^2}{2} + \frac{m\dot{y}^2}{2} =$$

$$E_K = \frac{m\dot{x}^2}{2} + \frac{1}{2}\frac{mr^2}{2}\left(\frac{\dot{x}}{r}\right)^2 + \left[\frac{1}{2}mR^2\left(\frac{\dot{x}-\dot{y}}{2R}\right)^2 + m\left(\frac{\dot{x}+\dot{y}}{2}\right)^2\right] + \frac{1}{2}\frac{mr^2}{2}\left(\frac{\dot{y}}{r}\right)^2 + \frac{m\dot{y}^2}{2}$$

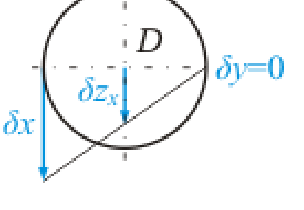
$$E_K = \frac{9}{8}m\dot{x}^2 + \frac{1}{4}\dot{x}\dot{y} + \frac{9}{8}m\dot{y}^2$$

Elementarni radovi i generalisane sile:

$$\delta A_x = G_D\,\delta z_x - T_A\,\delta x = 2mg\delta z_x - mg\mu_1\delta x$$

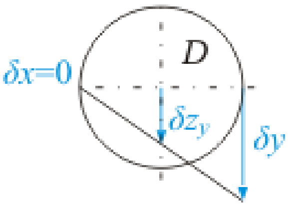
$$\delta A_x = 2mg\frac{\delta x}{2} - \frac{1}{12}mg\delta x$$

$$\delta A_x = \frac{11}{12}m\,g\,\delta x$$



$$Q_x = \frac{\delta A_x}{\delta x} = \frac{11}{12}m\,g$$

$$\delta A_y = G_D\,\delta z_y - G_E \sin 30^\circ \delta y - T_E\,\delta y$$
$$\delta A_y = 2mg\delta z_y - mg\sin 30^\circ \delta y - mg\cos 30^\circ \mu_2 \delta y$$
$$\delta A_y = 2mg\frac{\delta y}{2} - \frac{1}{2}mg\delta y - \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{12}mg\delta y$$
$$\delta A_y = \frac{3}{8}m\,g\,\delta y$$



$$Q_y = \frac{\delta A_y}{\delta y} = \frac{3}{8}m\,g$$

Lagranžove jednačine:

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{x}}\right) - \frac{\partial E_K}{\partial x} = Q_x \qquad \frac{9}{4}m\ddot{x} + \frac{1}{4}m\ddot{y} = \frac{11}{12}mg$$
$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{y}}\right) - \frac{\partial E_K}{\partial y} = Q_y \qquad \frac{1}{4}m\ddot{x} + \frac{9}{4}m\ddot{y} = \frac{3}{8}mg$$

$$\ddot{x} = \frac{63}{160}g = 3{,}863\,\frac{\mathrm{m}}{\mathrm{s}^2}$$
$$\ddot{y} = \frac{59}{480}g = 1{,}206\,\frac{\mathrm{m}}{\mathrm{s}^2}$$