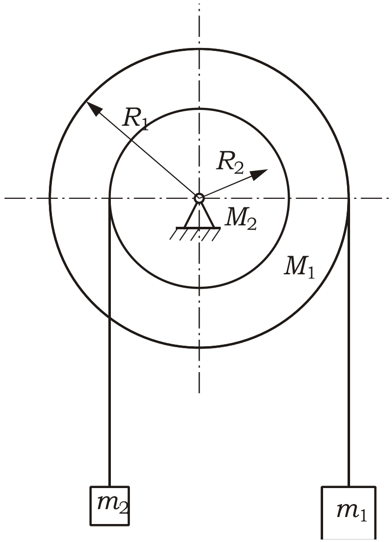
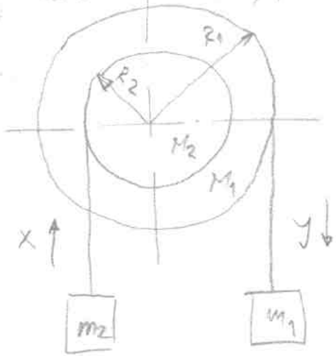
Dva tereta, masa m_1 i m_2 (m_1=4m_2=m), obješena su o dva laka nerastegljiva užeta, koji su obavijeni oko točkova poluprečnika R_1 i R_2, $R_1=\frac{3}{2}R_2$, masa M_1=M_2=$m/2$, prema slici. Točkovi su međusobno kruto spojeni i mogu se obrtati oko zajedničke horiznotalne ose *O*. Odrediti ugaono ubrzanje točkova smatrajući ih homogenim diskovima. Trenja zanemariti.



2. $\varphi, \dot{\varphi}, \ddot{\varphi}$



$m_1 = m$

$m_2 = \frac{m}{4}$

$R_1 = \frac{3}{2} R_2$

$M_1 = M_2 = \frac{m}{2}$

Kinematske veze

$dy = R_1 \cdot d\varphi \qquad dx = R_2 \cdot d\varphi = \frac{2}{3} R_1 \cdot d\varphi$

$\dot{y} = R_1 \cdot \dot{\varphi} \qquad \dot{x} = R_2 \cdot \dot{\varphi} = \frac{2}{3} R_1 \cdot \dot{\varphi}$

$\ddot{y} = R_1 \cdot \ddot{\varphi} \qquad \ddot{x} = R_2 \cdot \ddot{\varphi} = \frac{2}{3} \cdot R_1 \cdot \ddot{\varphi}$

Zakon o promjeni kinet. energ.

$$\frac{dE_k}{dt} = \frac{dA}{dt} \quad (A)$$

$$E_k = E_{k1} + E_{k2} + E_{kI} + E_{kII}$$

$$E_{k1} = \frac{m_1 \dot{y}^2}{2} = \frac{m (R_1 \cdot \dot{\varphi})^2}{2} = \frac{m R_1^2 \dot{\varphi}^2}{2}$$

$$E_{k2} = \frac{m_2 \dot{x}^2}{2} = \frac{\frac{m}{4} \left(\frac{2}{3} R_1 \dot{\varphi}\right)^2}{2} = \frac{m R_1^2 \dot{\varphi}^2}{18}$$

$$E_{kI} = \frac{J_I \dot{\varphi}^2}{2} = \frac{1}{2} \left(\frac{M_1 R_1^2}{2}\right) \dot{\varphi}^2 = \frac{1}{2} \left(\frac{\frac{m}{2} R_1^2}{2}\right) \dot{\varphi}^2 = \frac{m R_1^2 \dot{\varphi}^2}{8}$$

$$E_{kII} = \frac{J_{II} \dot{\varphi}^2}{2} = \frac{1}{2} \left(\frac{M_2 R_2^2}{2}\right) \dot{\varphi}^2 = \frac{1}{2} \frac{\frac{m}{2} \left(\frac{2}{3} R_1\right)^2}{2} \dot{\varphi}^2 =$$

$$E_{kII} = \frac{m R_1^2 \dot{\varphi}^2}{18}$$

$$E_k = \left(\frac{m R_1^2}{2} + \frac{m R_1^2}{18} + \frac{m R_1^2}{8} + \frac{m R_1^2}{18}\right) \dot{\varphi}^2 =$$

$$= \frac{36 + 4 + 9 + 4}{72} m R_1^2 \dot{\varphi}^2 = \frac{53}{72} m R_1^2 \dot{\varphi}^2$$

$$\frac{dE_k}{dt} = \frac{106}{72} m R_1^2 \dot{\varphi} \ddot{\varphi} = \frac{53}{36} m R_1^2 \dot{\varphi} \ddot{\varphi} \quad (B)$$

$$dA = G_1 \cdot dy - G_2 \cdot dx = mg \cdot R_1 \cdot d\varphi - \frac{m}{4} g \cdot \frac{2}{3} R_1 d\varphi =$$

$$= mg R_1 d\varphi - \frac{m}{6} g R_1 d\varphi = \frac{5}{6} mg R_1 d\varphi$$

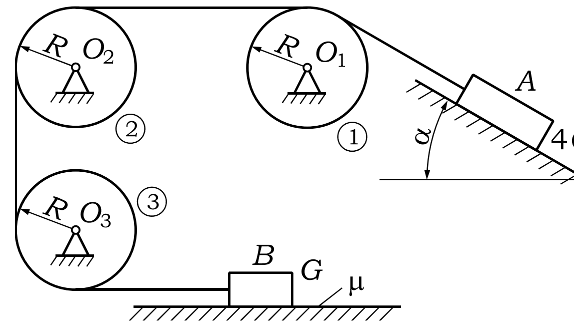
$$\frac{d\varphi}{dt} = \frac{5}{6} mg R_1 \dot{\varphi} \quad (C)$$

(B) i (C) u (A):

$$\frac{53}{36} \cancel{m} R_1^{\cancel{2}} \cancel{\dot{\varphi}} \ddot{\varphi} = \frac{5}{6} \cancel{m} g \cancel{R_1} \cancel{\dot{\varphi}}$$

$$\boxed{\ddot{\varphi}} = \frac{36 \cdot 5}{53 \cdot 6} \frac{1}{R_1} g = \boxed{\frac{30}{53} \frac{g}{R_1}}$$

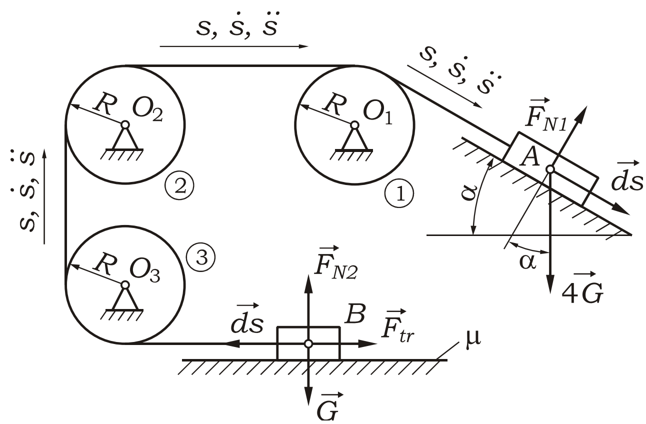
Po glatkoj strmoj ravni, s uglom nagiba $\alpha = 30°$, spušta se teret A težine 4G. Preko nepokretnih koturova 1,2 i 3 jednakih težina $\dfrac{G}{3}$ i jednakih poluprečnika R prebačeno je lako nerastegljivo uže, čiji je jedan kraj vezan za teret B, težina G, koji klizi po horizontalnoj hrapavoj ravni, koeficijenta trenja $\mu = \dfrac{1}{3}$. Odrediti ubrzanja tereta ako su koturovi 1,2 i 3 oblika diskova.



3.

Za rješavanje zadatak koristit ćemo se zakonom o promjeni kinetičke energije sistema u obliku:

$$\frac{dE_K}{dt} = \frac{dA}{dt} \text{.........................(A)}$$



Kinetičku energiju sistema čine energije koturova i tereta:

$$E_K = E_{K1} + E_{K2} + E_{K3} + E_{KA} + E_{KB}$$

Pošto su koturovi identični i imaju isto kretanje vrijedi:

$$E_K = 3 \cdot E_{K1} + E_{KA} + E_{KB}$$

Koturovi vrše rotaciju, a tereti translaciju pa je kinetička energija sistema:

$$E_K = 3 \cdot \frac{I_1 \cdot \omega_1^{\;2}}{2} + \frac{m_A \cdot \upsilon_A^{\;2}}{2} + \frac{m_B \cdot \upsilon_B^{\;2}}{2}$$

Ako uvrstimo kinematske karakteristike kretanja za sva tijela u sistemu dobijamo:

$$E_K = 3 \cdot \left[\frac{1}{2} \cdot \frac{m_1 \cdot R^2}{2} \cdot \left(\frac{\dot{s}}{R} \right)^2 \right] + \frac{m_A \cdot \dot{s}^2}{2} + \frac{m_B \cdot \dot{s}^2}{2}$$

Konačni izraz dobijamo uvrštavanjem zadanih težina tijela:

$$E_K = \frac{G}{4g} \cdot \dot{s}^2 + \frac{2 \cdot G \cdot \dot{s}^2}{g} + \frac{G \cdot \dot{s}^2}{2g} = \frac{G + 8 \cdot G + 2 \cdot G}{4g} \cdot \dot{s}^2$$

$$E_K = \frac{11}{4 \cdot g} \cdot G \cdot \dot{s}^2 \text{...(B)}$$

Elementarni rad dobijamo uzimanjem u obzir težine tereta A i sile trenja koja djeluje na teret B koje vrše rad pri elementarnom pomjeranju $\overrightarrow{ds}$:

$$dA = \overrightarrow{4G} \cdot \overrightarrow{ds} + \overrightarrow{F_{tr}} \cdot \overrightarrow{ds} = 4 \cdot G \cdot \sin\alpha \cdot ds - G \cdot \mu \cdot ds$$

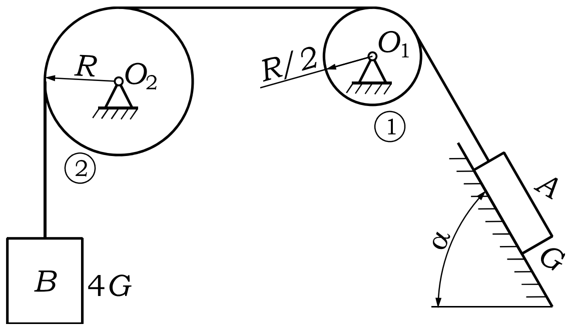
$$dA = 4 \cdot G \cdot \frac{1}{2} \cdot ds - G \cdot \frac{1}{3} \cdot ds = \frac{5}{3} \cdot G \cdot ds \text{..(C)}$$

(C) i (B) u (A):

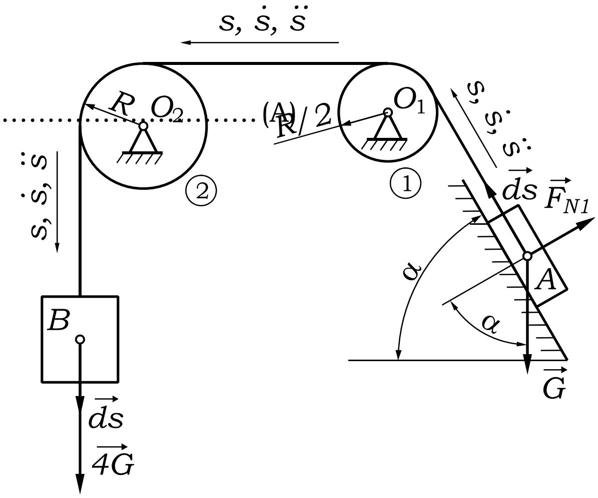
$$\frac{11}{2 \cdot g} \cdot G \cdot \dot{s} \cdot \ddot{s} = \frac{5}{3} \cdot G \cdot \dot{s}$$

$$\ddot{s} = \frac{10}{33} g$$

Po glatkoj strmoj ravni, s uglom nagiba $\alpha = 60°$, diže se teret A težine G. Preko nepokretnih koturova 1 i 2 težina $\frac{G}{2}$ i $\frac{G}{4}$, te poluprečnika $\frac{R}{2}$ i R prebačeno je lako nerastegljivo uže, čiji je jedan kraj vezan za teret A, a drugi za kraj za teret B težine 4G, koji se spušta vertikalno naniže. Odrediti ubrzanja tereta ako su koturovi 1 i 2 oblika diskova. Sva trenja zanemariti.



Rješenje:



Za rješavanje zadatak koristit ćemo se zakonom o promjeni kinetičke energije sistema u obliku:

$$\frac{dE_K}{dt} = \frac{dA}{dt} \quad \text{.........................(A)}$$

Kinetičku energiju sistema čine energije koturova i tereta:

$$E_K = E_{K1} + E_{K2} + E_{KA} + E_{KB}$$

Koturovi vrše rotaciju, a tereti translaciju pa je kinetička energija sistema:

$$E_K = \frac{I_1 \cdot \omega_1^2}{2} + \frac{I_2 \cdot \omega_2^2}{2} + \frac{m_A \cdot v_A^2}{2} + \frac{m_B \cdot v_B^2}{2}$$

Ako uvrstimo kinematske karakteristike kretanja za sva tijela u sistemu, te momente inercije koturova koje smatramo diskovima, dobijamo:

$$E_K = \frac{1}{2} \cdot \frac{m_1 \cdot \left(\frac{R}{2}\right)^2}{2} \cdot \left(\frac{\dot{s}}{\left(\frac{R}{2}\right)}\right)^2 + \frac{1}{2} \cdot \frac{m_2 \cdot R^2}{2} \cdot \left(\frac{\dot{s}}{R}\right)^2 + \frac{m_A \cdot \dot{s}^2}{2} + \frac{m_B \cdot \dot{s}^2}{2}$$

$$E_K = \frac{1}{4} \cdot m_1 \cdot \dot{s}^2 + \frac{1}{4} \cdot m_2 \cdot \dot{s}^2 + \frac{m_A \cdot \dot{s}^2}{2} + \frac{m_B \cdot \dot{s}^2}{2}$$

Konačni izraz dobijamo uvrštavanjem zadanih težina tijela:

$$E_K = \frac{G}{8g} \cdot s^2 + \frac{G}{16g} \cdot \dot{s}^2 + \frac{G \cdot \dot{s}^2}{2g} + 2\frac{G}{g} \cdot \dot{s}^2$$

$$E_K = \frac{2+1+8+32}{16} \frac{G}{g} \cdot \dot{s}^2 = \frac{43}{16} \frac{G}{g} \cdot \dot{s}^2 \quad \text{.........................(B)}$$

Elementarni rad dobijamo uzimanjem u obzir težine tereta A i B koje vrše rad pri elementarnom pomjeranju $\overrightarrow{ds}$:

$$dA = \overrightarrow{4G} \cdot \overrightarrow{ds} + \overrightarrow{G} \cdot \overrightarrow{ds} = 4 \cdot G \cdot ds - G \cdot \sin\alpha \cdot ds$$

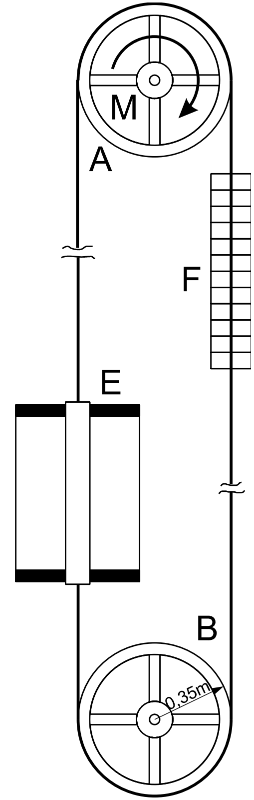
$$dA = 4 \cdot G \cdot ds - G \cdot \frac{\sqrt{3}}{2} \cdot ds = \frac{8-\sqrt{3}}{2} G \cdot ds \quad \text{.........................(C)}$$

(C) i (B) u (A):

$$\frac{43}{8 \cdot g} \cdot G \cdot \dot{s} \cdot \ddot{s} = \frac{8-\sqrt{3}}{2} \cdot G \cdot \dot{s}$$

$$\ddot{s} = \frac{32 - 4\sqrt{3}}{43} g = 0{,}583\ g = 5{,}72\ \frac{\mathrm{m}}{\mathrm{s}^2}$$

Kabina lifta E ima masu od 1,8 t, a protuteg F 2,3 t. Ukoliko motor pri pokretanju lifta pokreće pogonski kotur konstantnim momentom od 100 Nm, odrediti ubrzanje kabine lifta. Također odrediti vrijeme potrebno da kabina dostigne brzinu od 10 m/s iz stanja mirovanja. Koturovi A i B imaju masu od 150 kg i polupреčnik inercije $i = 0{,}2$ m. Zanemariti masu kablova i smatrati da ne proklizavaju.



$$\frac{dE_K}{dt} = \frac{dA}{dt}$$

$$E_K = E_{KA} + E_{KB} + E_{KE} + E_{KF}$$

$$E_K = 2\cdot\left(\frac{1}{2}\cdot 150\cdot 0{,}2^2\cdot\dot{\varphi}^2\right) + \frac{1}{2}\cdot 1{,}8\cdot 10^3\cdot(0{,}35\cdot\dot{\varphi})^2 + \frac{1}{2}\cdot 2{,}3\cdot 10^3\cdot(0{,}35\cdot\dot{\varphi})^2$$

$$E_K = 254{,}125\,\dot{\varphi}^2$$

$$dA = M\cdot d\varphi + G_F\cdot 0{,}35\cdot d\varphi - G_E\cdot 0{,}35\cdot d\varphi$$

$$dA = 100\cdot d\varphi + 2{,}3\cdot 10^3\cdot 9{,}81\cdot 0{,}35\,d\varphi - 1{,}8\cdot 10^3\cdot 9{,}81\cdot 0{,}35\,d\varphi$$

$$dA = 1816{,}75\ d\varphi$$

$$\frac{dE_K}{dt} = 508{,}25\,\dot{\varphi}\,\ddot{\varphi}$$

$$\frac{dA}{dt} = 1816{,}75\,\dot{\varphi}$$

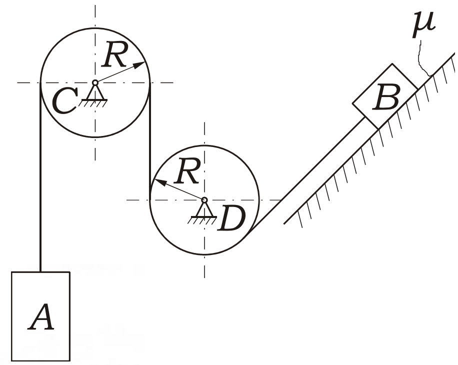
$$508{,}25\,\dot{\varphi}\,\ddot{\varphi} = 1816{,}75\,\dot{\varphi}$$

$$\ddot{\varphi} = \frac{1816{,}75}{508{,}25} = 3{,}575\,\mathrm{s}^{-2}$$

$$\boxed{a = \ddot{\varphi}\cdot 0{,}35 = 1{,}25\,\frac{\mathrm{m}}{\mathrm{s}^2}}$$

$$\boxed{t = \frac{\upsilon}{a} = \frac{10}{1{,}25} = 8\ \mathrm{s}}$$

Sistem tijela se sastoji od dva tereta A i B, masa $m_A = 4\ m_B$, $m_B = m$, povezanih užetom zanemarljive mase koje prelazi preko koturova C i D poluprečnika R, prema slici. Teret B klizi po strmoj ravni nagiba α uz koeficijent trenja μ. Smatrajući koturove homogenim diskovima masa $m_C = m_D = m$ izračunati ubrzanje tereta A.



$$E_K = E_{KA} + E_{KB} + E_{KC} + E_{KD}$$

$$= \frac{m_A v_A^2}{2} + \frac{m_D R_D^2}{2} \cdot \frac{\omega_D^2}{2} + \frac{m_C R_C^2}{2} \cdot \frac{\omega_C^2}{2} + \frac{m_B v_B^2}{2}$$

$$v_A = v_B = \dot{s} \qquad R_C = R_D = R \qquad m_C = m_D = m$$

$$\omega_C = \omega_D = \frac{\dot{s}}{R} \qquad m_A = 4m \qquad m_B = m$$

$$E_K = \frac{4m\dot{s}^2}{2} + \frac{m\cancel{R^2}}{4} \cdot \frac{\dot{s}^2}{\cancel{R^2}} + \frac{m\cancel{R^2}}{4} \cdot \frac{\dot{s}^2}{\cancel{R^2}} + \frac{m\dot{s}^2}{2}$$

$$E_K = 3m\dot{s}^2$$

$$dA = G_A \cdot ds + G_B \cdot \sin\alpha \cdot ds - F_t \cdot ds$$

$$F_t = N\mu = G_B \cdot \cos\alpha\,\mu \qquad \Sigma y = 0 \quad G_B \cdot \cos\alpha - N = 0$$

$$dA = 4m \cdot g\,ds + m\,g \cdot \sin\alpha\,ds - m \cdot g \cdot \cos\alpha\,\mu\,ds$$

$$= mg\,(4 + \sin\alpha - \cos\alpha\,\mu) \cdot ds$$

$$\frac{dE_k}{dt} = \frac{dA}{dt}$$

$$6\,\cancel{m}\,\cancel{\dot{s}}\,\ddot{s} = \cancel{m}g(4 + \sin\alpha - \cos\alpha\,\mu)\,\cancel{\dot{s}}$$

$$\ddot{s} = g\,\frac{4 + \sin\alpha - \cos\alpha\,\mu}{6}$$