Sample Review of FE (**F**undamental of **E**ngineering Examination)

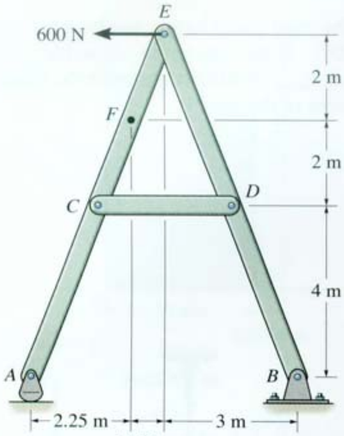
Mechanics of Materials, 6th Edition in SI Units
R.C.**Hibbeler**

ผศ.ทวีเดช ศิริธนาพิพัฒน์
สาขาไฟฟ้าเครื่องกลการผลิต ภาควิชาวิศวกรรมเครื่องกล
คณะวิศวกรรมศาสตร์ มหาวิทยาลัยเกษตรศาสตร์

Stress

Chapter 1—Review All Sections

D-1. Determine the resultant internal moment in the member of the frame at point F.



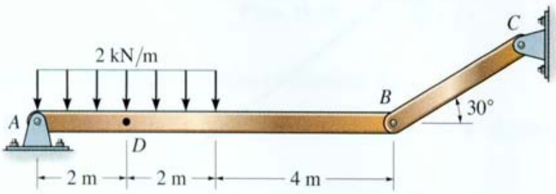
Prob. D–1

D-1. Entire frame:
$\Sigma M_B = 0;\quad A_y = 800$ N
CD is a two-force member
Member AE:
$\Sigma M_E = 0;\quad F_{CD} = 600$ N
Segment ACF:
$\Sigma M_F = 0;\quad M_F = 600$ N·m

D-2. The beam is supported by a pin at A and a link BC. Determine the resultant internal shear in the beam at point D.

D-3. The beam is supported by a pin at A and a link BC. Determine the average shear stress in the pin at B if it has a diameter of 20 mm and is in double shear.

D-4. The beam is supported by a pin at A and a link BC. Determine the average shear stress in the pin at A if it has a diameter of 20 mm and is in single shear.



Probs. D–2/3/4

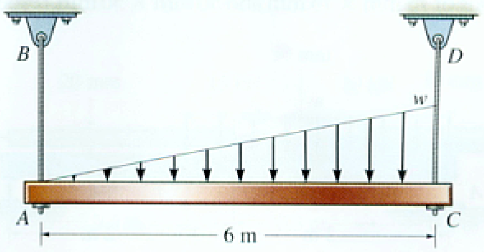
D-2. BC is a two-force member.
Beam AB:
$\Sigma M_B = 0;\quad A_y = 6$ kN
Segment AD:
$\Sigma F_y = 0;\quad V = 2$ kN *Ans.*

D-3. BC is a two-force member.
Beam AB:
$\Sigma M_A = 0;\quad T_{BC} = 4$ kN
Pin B:
$$\tau_B = \frac{T_{BC}/2}{A} = \frac{4/2}{\frac{\pi}{4}(0.02)^2} = 6.37 \text{ MPa}$$ *Ans.*

D-4. BC is a two-force member
Beam AB:
$\Sigma M_A = 0;\quad T_{BC} = 4$ kN
$\Sigma F_x = 0;\quad A_x = 3.464$ kN
$\Sigma F_y = 0;\quad A_y = 6$ kN
$$F_A = \sqrt{(3.464)^2 + (6)^2} = 6.928 \text{ kN}$$
$$\tau_A = \frac{F_A}{A} = \frac{6.928}{\frac{\pi}{4}(0.02)^2} = 22.1 \text{ MPa}$$ *Ans.*

Stress

D-8. The uniform beam is supported by two rods AB and CD that have cross-sectional areas of 10 mm^2 and 15 mm^2, respectively. Determine the intensity w of the distributed load so that the average normal stress in each rod does not exceed 300 kPa.



Prob. D–8

D-8. Beam:

$\Sigma M_A = 0;\quad T_{CD} = 2\,w$

$\Sigma F_y = 0;\quad T_{AB} = w$

Rod AB:

$\sigma = \frac{P}{A};\quad 300(10^3) = \frac{w}{10};$

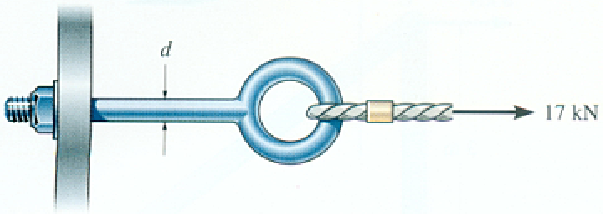
$w = 3$ MN/m

Rod CD:

$\sigma = \frac{P}{A};\quad 300(10^3) = \frac{2w}{15};$

$w = 2.25$ MN/m *Ans.*

D-9. The bolt is used to support the load of 17 kN. Determine its diameter d to the nearest mm. The allowable normal stress for the bolt is $\sigma_{\text{allow}} = 170$ MPa.



Prob. D–9

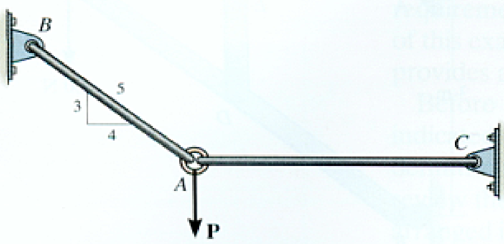
D-9. $\sigma = \frac{P}{A};\quad 170 = \frac{17 \times 10^3}{\frac{\pi}{4}d^2};$

$d = 11.3$ mm

use $d = 12$ mm *Ans.*

D-10. The two rods support the vertical force of $P = 30$ kN. Determine the diameter of rod AB if the allowable tensile stress for the material is $\sigma_{\text{allow}} = 150$ MPa.

D-11. The rods AB and AC have diameters of 15 mm and 12 mm, respectively. Determine the largest vertical force **P** that can be applied. The allowable tensile stress for the rods is $\sigma_{\text{allow}} = 150$ MPa.



Probs. D–10/11

D-10. Joint A:

$\Sigma F_y = 0;\quad F_{AB} = 50$ kN

$\sigma = \frac{P}{A};\quad 150(10^6) = \frac{50(10^3)}{\frac{\pi}{4}d^2};$

$d = 20.6$ mm *Ans.*

D-11. Joint A:

$\Sigma F_y = 0;\quad F_{AB} = 1.667P$

$\Sigma F_x = 0;\quad F_{AC} = 1.333P$

Road AB:

$\sigma = \frac{P}{A};\quad 150(10^6) = \frac{1.667P}{\frac{\pi}{4}(0.015)^2};$

$P = 15.9$ kN

Rod AC:

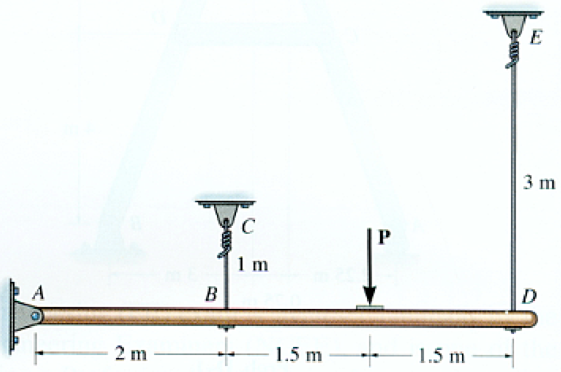
$\sigma = \frac{P}{A};\quad 150(10^6) = \frac{1.333P}{\frac{\pi}{4}(0.012)^2};$

$P = 12.7$ kN *Ans.*

Strain

D-13. A rubber band has an unstretched length of 180 mm. If it is stretched around a pole having a diameter of 60 mm, determine the average normal strain in the band.

D-14. The rigid rod is supported by a pin at A and wires BC and DE. If the maximum allowable normal strain in each wire is $\epsilon_{allow} = 0.003$, determine the maximum vertical displacement of the load **P**.



Prob. D–14

D-13. $\epsilon = \dfrac{l - l_0}{l_0} = \dfrac{\pi(60) \times 180}{180} = 0.0472 \text{ mm/mm}$ *Ans.*

D-14. $(\delta_{DE})_{max} = \epsilon_{max} L_{DE} = 0.003(3) = 0.009 \text{ m}$

By proportion from A,

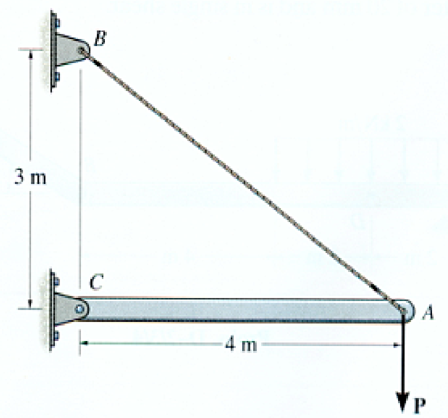
$\delta_{BC} = 0.009\left(\frac{2}{5}\right) = 0.0036 \text{ m}$

$(\delta_{BC})_{max} = \epsilon_{max} l_{BC} = 0.003(1) = 0.003 \text{ m} < 0.0036 \text{ m}$

Use $\delta_{BC} = 0.003$ m. By proportion from A.

$\delta_P = 0.003\left(\dfrac{3.5}{2}\right) = 0.00525 \text{ m} = 5.25 \text{ mm}$ *Ans.*

D-15. The load **P** causes a normal strain of 0.0045 mm/mm in cable AB. Determine the angle of rotation of the rigid beam due to the loading if the beam is originally horizontal before it is loaded.



Prob. D–15

D-15. $l_{AB} = \sqrt{(4)^2 + (3)^2} = 5 \text{ m}$

$l'_{AB} = 5 + 5(0.0045) = 5.0225 \text{ m}$

The angle BCA was originally $\theta = 90°$. Using the cosine law, the new angle BCA (θ') is

$5.0225 = \sqrt{(3)^2 + (4)^2 - 2(3)(4)\cos\theta}$

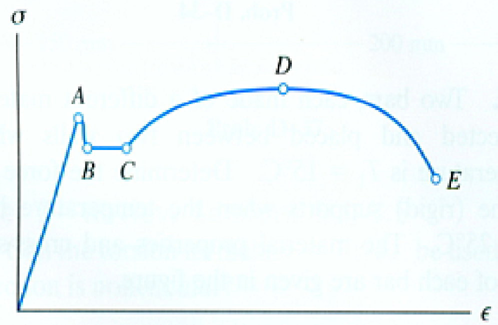
$\theta = 90.538°$

Thus,

$\Delta\theta' = 90.538° - 90° = 0.538°$ *Ans.*

D-17. Define homogeneous material.

D-18. Indicate the points on the stress-strain diagram which represent the proportional limit and the ultimate stress.



Prob. D–18

D-17. Material has uniform properties throughout. *Ans.*
D-18. Proportional limit is *A*. *Ans.*
Ultimate stress is *D*. *Ans.*

D-19. Define the modulus of elasticity *E*.

D-20. At room temperature, mild steel is a ductile material. True or false?

D-21. Engineering stress and strain are calculated using the *actual* cross-sectional area and length of the specimen. True or false?

D-22. If a rod is subjected to an axial load, there is only strain in the material in the direction of the load. True or false?

D-19. The initial slope of the $\sigma - \epsilon$ diagram. *Ans.*
D-20. True. *Ans.*
D-21. False. Use the *original* cross-sectional area and length. *Ans.*
D-22. False. There is also strain in the perpendicular directions due to the Poisson effect. *Ans.*

D-23. A 100-mm long rod has a diameter of 15 mm. If an axial tensile load of 100 kN is applied, determine its change in length. $E = 200$ GPa.

D-23. $\epsilon = \frac{\sigma}{E} = \epsilon\frac{P}{AE}$

$$\delta = \epsilon L = \frac{PL}{AE} = \frac{100(10^3)(0.100)}{\frac{\pi}{4}(0.015)^2\,200(10^9)} = 0.283\text{ mm} \qquad \textit{Ans.}$$

D-24. A bar has a length of 200 mm and cross-sectional area of 7500 mm^2. Determine the modulus of elasticity of the material if it is subjected to an axial tensile load of 50 kN and stretches 0.075 mm. The material has linear-elastic behavior.

D-25. A 10-mm-diameter brass rod has a modulus of elasticity of $E = 100$ GPa. If it is 4 m long and subjected to an axial tensile load of 6 kN, determine its elongation.

D-26. A 100-mm-long rod has a diameter of 15 mm. If an axial tensile load of 10 kN is applied to it, determine its change in diameter. $E = 70$ GPa, $\nu = 0.35$.

D-24. $\epsilon = \frac{\sigma}{E} = \frac{P}{AE}$

$$\delta = \epsilon L = \frac{PL}{AE};$$

$$0.075 = \frac{50 \times 10^3 \times (200)}{7500E}$$

$$E = 17.78\ (10^3)\text{ MPa} \qquad \textit{Ans.}$$

D-25. $\epsilon = \frac{\sigma}{E} = \frac{P}{AE}$

$$\delta = \epsilon L = \frac{PL}{AE} = \frac{6(10^3)4}{\frac{\pi}{4}(0.01)^2 100(10^9)} = 3.06\text{ mm} \qquad \textit{Ans.}$$

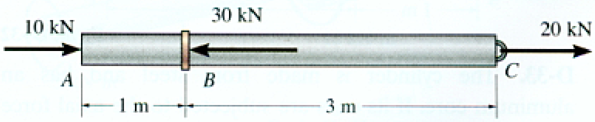
D-26. $\sigma = \frac{P}{A} = \frac{10(10^3)}{\frac{\pi}{4}(0.015)^2} = 56.59$ MPa

$$\epsilon_{long} = \frac{\sigma}{E} = \frac{56.59(10^6)}{70(10^9)} = 0.808(10^{-3})$$

$$\epsilon_{lat} = -\nu\epsilon_{long} = -0.35(0.808(10^{-3})) = -0.283(10^{-3})$$

$$\delta d = \epsilon_{lat}(15\text{ mm}) = -4.24\text{ mm} \qquad \textit{Ans.}$$

D-29. Determine the displacement of end A with respect to end C of the shaft. The cross-sectional area is 300 mm^2 and $E = 210(10^3)$ MPa.

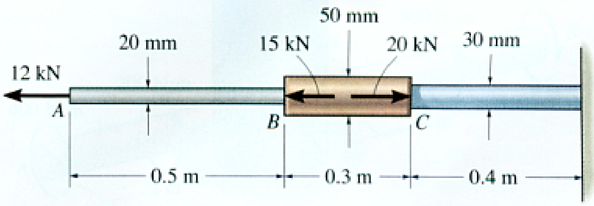


Prob. D-29

D-29. $\delta_{A/C} = \sum \frac{PL}{AE} = \frac{-10 \times 10^3 \times 10^3}{300(210)(10^3)} + \frac{20 \times 10^3 \times 3 \times 10^3}{300(210)(10^3)}$

$= 0.794$ mm *Ans.*

D-30. Determine the displacement of end A with respect to C of the shaft. The diameters of each segment are indicated in the figure. $E = 200$ GPa.

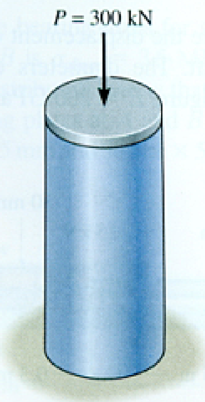


Prob. D-30

D-30. $\delta_{A/C} = \sum \frac{PL}{AE} = \frac{12(10^3)(0.5)}{\frac{\pi}{4}(0.02)^2 200(10^9)}$

$+ \frac{27(10^3)(0.3)}{\frac{\pi}{4}(0.05)^2 200(10^9)} = 0.116$ mm *Ans.*

D-33. The cylinder is made from steel and has an aluminum core. If its ends are subjected to the axial force of 300 kN, determine the average normal stress in the steel. The cylinder has an outer diameter of 100 mm and an inner diameter of 80 mm. $E_{st} = 200$ GPa, $E_{al} = 73.1$ GPa.



Prob. D-33

D-33. Equilibrium:

$P_{st} + P_{al} = 300(10^3)$

Compatibility:

$\delta_{st} = \delta_{al};$

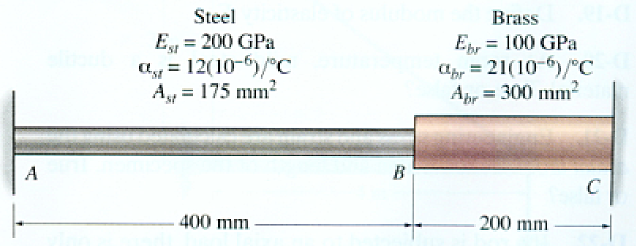
$$\frac{P_{st}L}{\left[\frac{\pi}{4}(0.1)^2 - \frac{\pi}{4}(0.08)^2\right]200(10^9)} = \frac{P_{al}L}{\left[\frac{\pi}{4}(0.08)^2\right]73.1(10^9)}$$

$P_{st} = 181.8$ kN

$P_{al} = 118$ kN

$\sigma_{st} = \frac{P_{st}}{A} = \frac{181.8}{\left[\frac{\pi}{4}(0.1)^2 - \frac{\pi}{4}(0.08)^2\right]} = 64.3$ MPa *Ans.*

D-35. Two bars, each made of a different material, are connected and placed between two walls when the temperature is $T_1 = 15°C$. Determine the force exerted on the (rigid) supports when the temperature becomes $T_2 = 25°C$. The material properties and cross-sectional area of each bar are given in the figure.



Prob. D–35

D-35.

$$\delta_{temp} = \Sigma\, \alpha \Delta T L$$

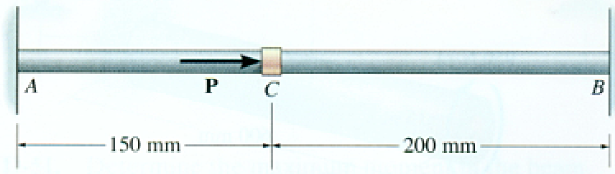
$$\delta_{load} = \Sigma \frac{PL}{AE}$$

Compatibility: $\delta_{temp} + \delta_{load} = 0$

$$12(10^{-6})(25-15)(0.4) + 21(10^{-6})(25-15)(0.2) + \frac{-F(0.4)}{175(10^{-6})(200(10^9))} - \frac{F(0.2)}{300(10^{-6})(100(10^9))} = 0$$

$P = 4.97$ kN *Ans.*

D-36. The aluminum rod has a diameter of 10 mm and is attached to the rigid supports at A and B when $T_1 = 80°C$. If the temperature becomes $T_2 = 100°C$, and an axial force of $P = 6000$ N is applied to the rigid collar as shown, determine the reactions at A and B. $\alpha_{al} = 20(10^{-6})/°C$, $E_{al} = 75(10^3)$ MPa.



Prob. D–36

D-36. Equilibrium:

$F_A + F_B = 6000$

Compatibility

Remove support at B. Require

$$\delta_B = (\delta_{B/A})_{temp} + (\delta_{B/A})_{load} = 0$$

$$\alpha\, \Delta T L + \sum \frac{PL}{AE} = 0$$

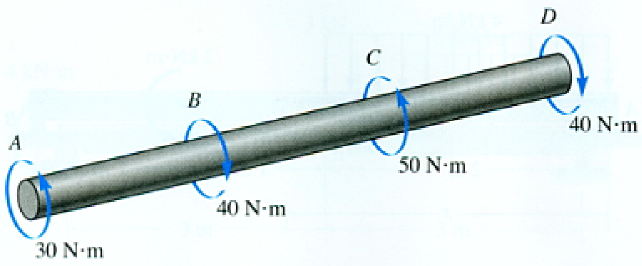
$$20(10^{-6})(100-80)(350) + \frac{6000(150)}{\frac{\pi}{4}(10)^2 75(10^3)} - \frac{F_B(350)}{\frac{\pi}{4}(10)^2 75(10^3)} = 0$$

$F_B = 4.93$ kN *Ans.*

$F_A = 1.07$ kN *Ans.*

D-38. Can the torsion formula, $\tau = Tc/J$, be used if the cross section is noncircular?

D-39. The solid 20-mm-diameter shaft is used to transmit the torques shown. Determine the absolute maximum shear stress developed in the shaft.



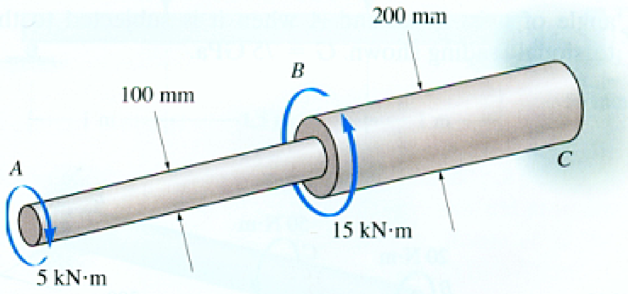
Prob. D–39

D-38. No, it is only valid for circular cross sections. Noncircular cross sections will warp. *Ans.*

D-39. $T_{max} = T_{CD} = 40$ N·m

$$\tau_{max} = \frac{Tc}{J} = \frac{40(10 \times 10^3)}{\frac{\pi}{2}(10)^4} = 25.46 \text{ MPa} \qquad \textit{Ans.}$$

D-41. The solid shaft is used to transmit the torques shown. Determine the absolute maximum shear stress developed in the shaft.



Prob. D–41

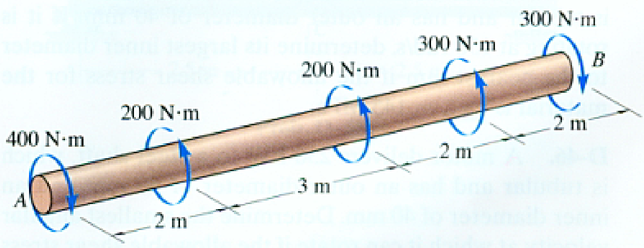
D-41. Segment AB:

$$\tau_{max} = \frac{Tc}{J} = \frac{5(10^3)(0.05)}{\frac{\pi}{2}(0.05)^4} = 25.5 \text{ MPa} \qquad \textit{Ans.}$$

Segment BC:

$$\tau_{max} = \frac{Tc}{J} = \frac{10(10^3)(0.1)}{\frac{\pi}{4}(0.1)^4} = 6.37 \text{ MPa}$$

D-42. The shaft is subjected to the torques shown. Determine the angle of twist of end A with respect to end B. The shaft has a diameter of 40 mm. $G = 80(10^3)$ MPa.

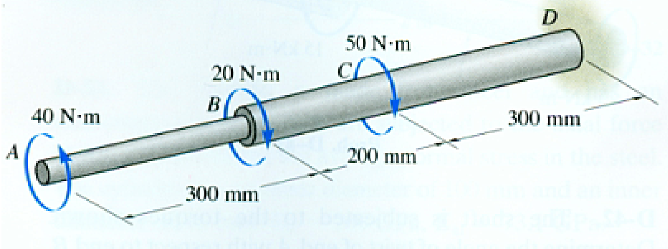


Prob. D–42

D-42. $$\phi_{A/B} = \sum \frac{TL}{JG} = \frac{-400(1000)(2)(1000)}{\frac{\pi}{2}(20)^4 80(10^3)} - \frac{200(1000)(3)(1000)}{\frac{\pi}{2}(20)^4 80(10^3)} + 0 + \frac{300(1000)(2)(1000)}{\frac{\pi}{2}(20)^4 80(10^3)}$$

$= -0.0399$ rad $= 0.0399$ rad clockwise when viewed from A. *Ans.*

D-44. The shaft consists of a solid section AB with a diameter of 30 mm, and a tube BD with an inner diameter of 25 mm and outer diameter of 50 mm. Determine the angle of twist at its end A when it is subjected to the torsional loading shown. $G = 75$ GPa.



Prob. D–44

D-44. $\phi_A = \sum \frac{TL}{JG} = \frac{40(0.3)}{\frac{\pi}{2}(0.015)^4 75(10^9)}$

$+ \frac{20(0.2)}{\frac{\pi}{2}[(0.025)^4 - (0.0125)^4]75(10^9)}$

$- \frac{30(0.3)}{\frac{\pi}{2}[(0.025)^4 - (0.0125)^4]75(10^9)}$

$= 1.90(10^{-3})$ rad counterclockwise when viewed from A towards D. *Ans.*

D-45. A motor delivers 150 kW to a steel shaft, which is tubular and has an outer diameter of 40 mm. If it is rotating at 150 rad/s, determine its largest inner diameter to the nearest mm if the allowable shear stress for the material is $\tau_{allow} = 145$ MPa.

D-46. A motor delivers 250 kW to a steel shaft, which is tubular and has an outer diameter of 50 mm and an inner diameter of 40 mm. Determine the smallest angular velocity at which it can rotate if the allowable shear stress for the material is $\tau_{allow} = 145$ MPa.

D-45. $T = \frac{P}{\omega} = \frac{150\,000}{150} = 1000$ N·m $= 10^6$ N·mm

$\tau_{allow} = \frac{Tc}{J};\quad 145 = \frac{1 \times 10^6 \times 20}{\frac{\pi}{2}[(20)^4 - r_i^4]}$

$r_i = 16.4$ mm

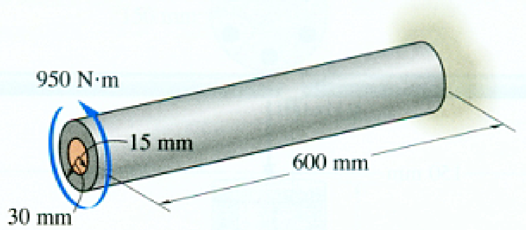
$d_i = 32.8$ mm use $d = 33$ mm *Ans.*

D-46. $T = \frac{P}{\omega} = \frac{250\,000}{\omega}$

$\tau_{max} = \frac{Tc}{J};\quad 145 = \frac{\frac{250\,000}{\omega}(10^3)(25)}{\frac{\pi}{2}[(25)^4 - (20)^4]}$

$\omega = 119$ rad/s *Ans.*

D-47. The shaft is made from a steel tube having a brass core. If it is fixed to the rigid support, determine the angle of twist that occurs at its end. $G_{st} = 75$ GPa and $G_{br} = 37$ GPa.



Prob. D–47

D-47. Equilibrium:

$T_{st} + T_{br} = 950$

Compatibility: $\phi_{st} = \phi_{br}$;

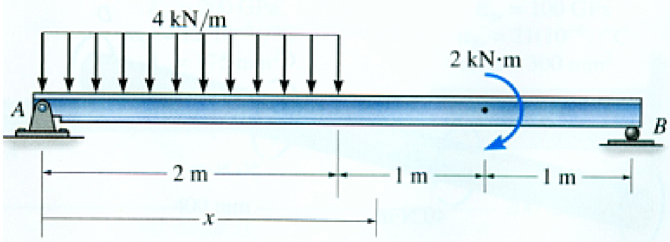
$\frac{T_{st}(0.6)}{\frac{\pi}{2}[(0.03)^4 - (0.015)^4]75(10^9)} = \frac{T_{br}(0.6)}{\frac{\pi}{2}(0.015)^4 37(10^9)}$

$T_{br} = 30.25$ N·m

$T_{st} = 919.8$ N·m

$\phi = \phi_{br} = \frac{30.25(0.6)}{\frac{\pi}{2}(0.015)^4 37(10^9)} = 0.00617$ rad *Ans.*

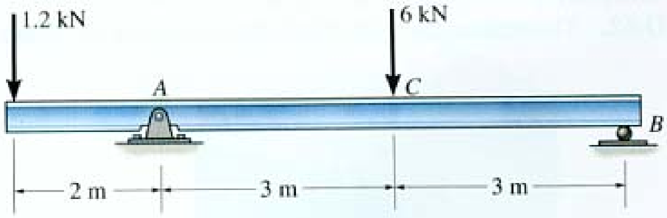
D-49. Determine the internal moment in the beam as a function of x, where $2\text{ m} \le x < 3\text{ m}$.



Prob. D–49

D-49. $A_y = 5.5$ kN
Use section of length x.
$\circlearrowleft + \Sigma M = 0;\ -5.5x + 4(2)(x - 1) + M = 0$
$M = 8 - 2.5x$ *Ans.*

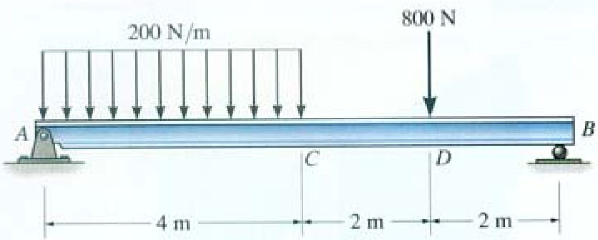
D-51. Determine the maximum moment in the beam.



Prob. D–51

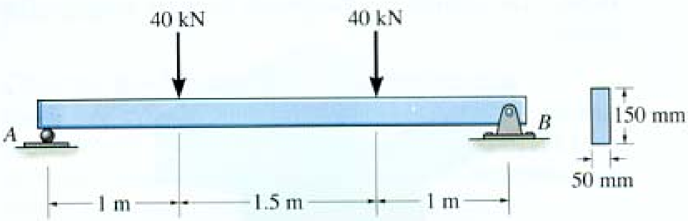
D-51. $B_y = 2.6$ kN
$A_y = 4.6$ kN
Draw M-diagram
$M_{max} = 7.80\ \text{kN} \cdot \text{m}$ (at C) *Ans.*

D-54. Determine the maximum moment in the beam.



Prob. D–54

D-55. Determine the absolute maximum bending stress in the beam.



Prob. D–55

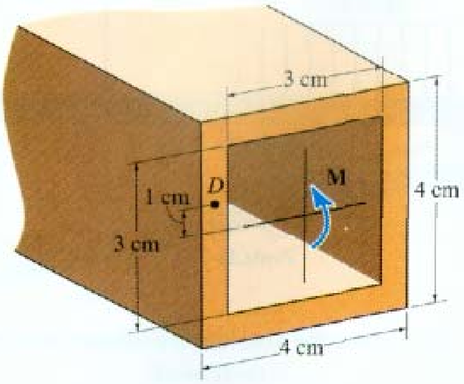
D-54. $A_y = B_y = 800$ N
Draw M-diagram
$M_{max} = 1600\ \text{N} \cdot \text{m}$ (within CD) *Ans.*

D-55. $A_y = B_y = 40$ kN
$M_{max} = 40(1) = 40$ kN·m
$\sigma = \frac{Mc}{I} = \frac{40(10^6)(75)}{\frac{1}{12}(50)(150)^3} = 213.3\text{ MPa}$ *Ans.*

D-57. What is the strain in a beam at the neutral axis?

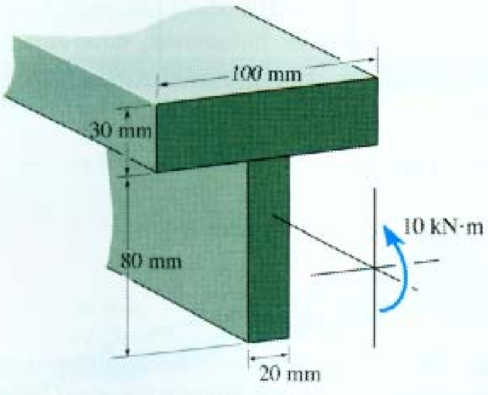
D-57. $\epsilon = 0$ *Ans.*

D-58. Determine the moment M that should be applied to the beam in order to create a compressive stress at point D of 10 N/mm².



Prob. D–58

D-59. Determine the maximum bending stress in the beam.



Prob. D–59

D-58. $\sigma = \frac{My}{I}; \quad 10(10^2) = \frac{M(1)}{\left[\frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3\right]}$

$M = 14.58 \text{ kN·cm} = 145.8 \text{ N·m}$ *Ans.*

D-59. From bottom of cross section,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{40(80)(20) + 95(30)(100)}{80(20) + 30(100)} = 75.870 \text{ mm}$$

$$I = \frac{1}{12}(20)(80)^3 + 20(80)(75.870 - 40)^2$$

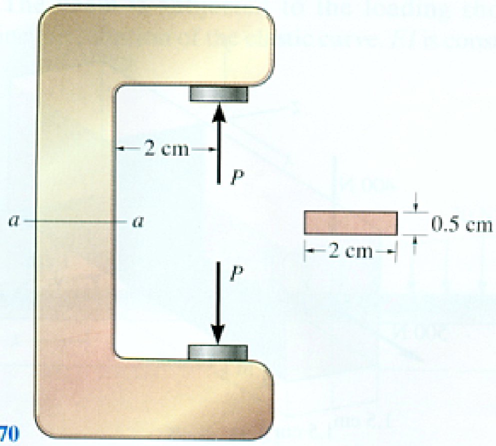
$$+\frac{1}{12}(100)(30)^3 + 100(30)(95 - 75.870)^2 = 4.235(10^{-6})\text{ m}$$

$$\sigma_{\max} = \frac{Mc}{I} = \frac{10(10^3)(0.075870)}{4.235(10^{-6})} = 179 \text{ MPa}$$ *Ans.*

D-68. A cylindrical tank is subjected to an internal pressure of 80 N/cm^2. If the internal diameter of the tank is 30 cm, and the wall thickness is 0.3 cm, determine the maximum normal stress in the material.

D-69. A pressurized spherical tank is to be made of 0.25-cm-thick steel. If it is subjected to an internal pressure of p = 150 N/cm^2, determine its inner diameter if the maximum normal stress is not to exceed $10(10^3)$ N/cm^2.

D-70. Determine the magnitude of the load P that will cause a maximum normal stress of σ_{max} = 30 N/cm^2 in the link along section a–a.



Prob. D–70

D-68. $\sigma = \dfrac{pr}{t} = \dfrac{80(15)}{0.3} = 4000\ \text{N/cm} = 40\ \text{MPa}$ *Ans.*

D-69. $\sigma = \dfrac{pr}{2t};\quad 10(10^3) = \dfrac{150r}{2(0.25)}$

$r = 33.3$ cm

$d = 66.7$ cm *Ans.*

D-70. At the section through centroidal axis,

$N = P$

$V = 0$

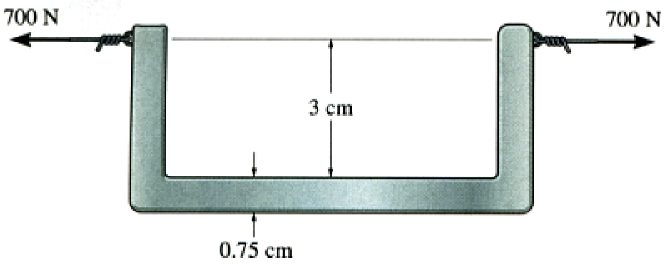
$M = (2 + 1)P = 3P$

$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

$$30 = \frac{P}{2(0.5)} + \frac{(3P)(1)}{\frac{1}{12}(0.5)(2)^3}$$

$P = 3$ N *Ans.*

D-71. Determine the maximum normal stress in the horizontal portion of the bracket. The bracket has a thickness of 1 cm and a width of 0.75 cm.



Prob. D–71

D-71. At a section through the center of bracket on centroidal axis.

$N = 700\text{ N}$

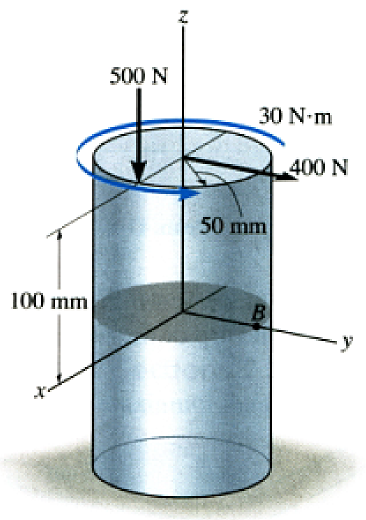
$V = 0$

$M = 700(3 + 0.375) = 2362.5\text{ N}\cdot\text{cm}$

$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{700}{0.75(1)} + \frac{2362.5(0.375)}{\left[\frac{1}{12}(1)(0.75)^3\right]}$$

$= 26.1\text{ N/cm}^2 = 0.261\text{ MPa}$ *Ans.*

D-74. The solid cylinder is subjected to the loading shown. Determine the components of stress at point *B*.



Prob. D–74

D-74. At section *B*:

$N_z = 500\text{ N},\ V_y = 400\text{ N},$

$M_x = 400(0.1) = 40\text{ N}\cdot\text{m}$

$M_y = 500(0.05) = 25\text{ N}\cdot\text{m},\ T_z = 30\text{ N}\cdot\text{m}$

Axial load:

$$\sigma_z = \frac{P}{A} = \frac{500}{\pi(0.05)^2} = 63.66\text{ kPa (C)}$$

Shear load:

$\tau_{zy} = 0$ since at B, $Q = 0$.

Moment about x axis:

$$\sigma_z = \frac{Mc}{I} = \frac{40(0.05)}{\frac{\pi}{4}(0.05)^4} = 407.4\text{ kPa (C)}$$

Moment about y axis:

$\sigma_z = 0$ since B is on neutral axis.

Torque:

$$\tau_{zx} = \frac{Tc}{J} = \frac{30(0.05)}{\frac{\pi}{2}(0.05)^4} = 153\text{ kPa}$$

Thus,

$\sigma_x = 0$ *Ans.*

$\sigma_y = 0$ *Ans.*

$\sigma_z = -63.66 - 407.4 = -471\text{ kPa}$ *Ans.*

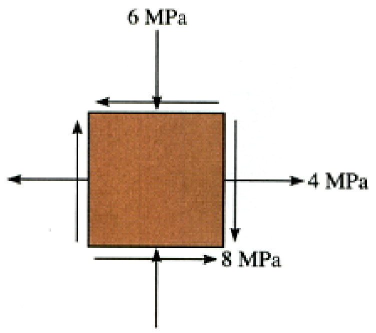
$\tau_{xy} = 0$ *Ans.*

$\tau_{zy} = 0$ *Ans.*

$\tau_{zx} = -153\text{ kPa}$ *Ans.*

D-75. When the state of stress at a point is represented by the principal stress, no shear stress will act on the element. True or false?

D-76. The state of stress at a point is shown on the element. Determine the maximum principal stress.

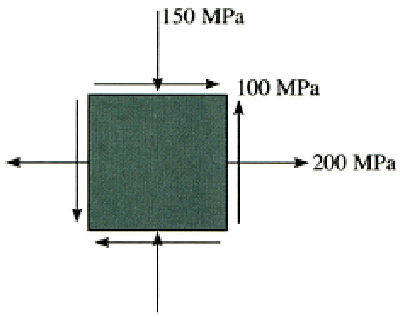


Prob. D–76

D-75. True. *Ans.*

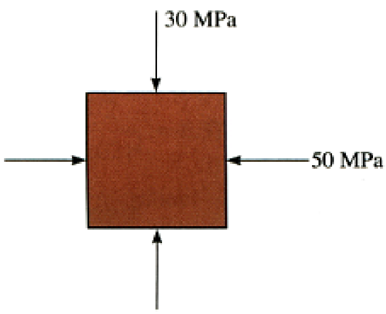
D-76. $\sigma_y = 4$ MPa, $\sigma_y = -6$ MPa, $\tau_{xy} = -8$ MPa
Apply Eq. 9-5, $\sigma_1 = 8.43$ MPa
$\sigma_2 = -10.4$ MPa *Ans.*

D-77. The state of stress at a point is shown on the element. Determine the maximum in-plane shear stress.



Prob. D–77

D-78. The state of stress at a point is shown on the element. Determine the maximum in-plane shear stress.

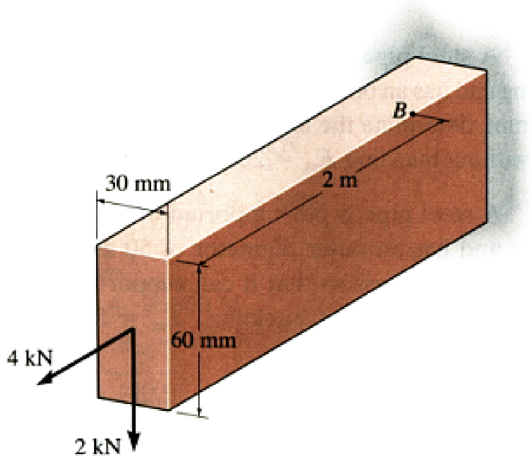


Prob. D–78

D-77. $\sigma_y = 200$ MPa, $\sigma_y = -150$ MPa, $\tau_{xy} = 100$ MPa
Apply Eq. 9-7, $\tau_{\substack{max \\ in\text{-}plane}} = 202$ MPa *Ans.*

D-78. $\sigma_x = -50$ MPa, $\sigma_y = -30$ MPa, $\tau_{xy} = 0$
Use Eq. 9-7, $\tau_{\substack{max \\ in\text{-}plane}} = 10$ MPa *Ans.*

D-79. The beam is subjected to the load at its end. Determine the maximum principal stress at point B.



Prob. D–79

D-79. At the cross section through B:

$N = 4\text{ kN},\ V = 2\text{ kN},\ M = 2(2) = 4\text{ kN}\cdot\text{m}$

$$\sigma_B = \frac{P}{A} + \frac{Mc}{I} = \frac{4(10^3)}{0.03(0.06)} + \frac{4(10^3)(0.03)}{\frac{1}{12}(0.03)(0.06)^3}$$

$= 224\text{ MPa (T)}$

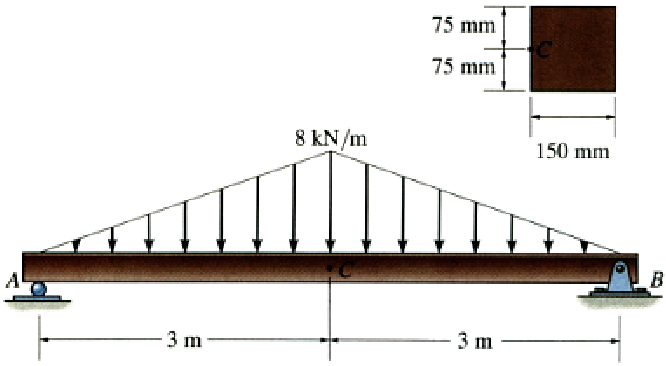
Note $\tau_B = 0$ since $Q = 0$.

Thus,

$\sigma_1 = 224\text{ MPa}$ *Ans.*

$\sigma_2 = 0$ *Ans.*

D-80. The beam is subjected to the loading shown. Determine the principal stress at point C.



Prob. D–80

D-80. $A_y = B_y = 12\text{ kN}$

Segment AC:

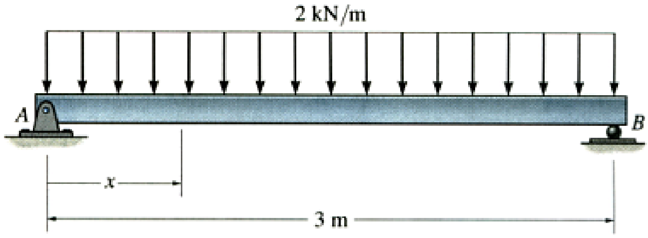
$V_C = 0,\ M_C = 24\text{ kN}\cdot\text{m}$

$\tau_C = 0$ (since $V_C = 0$)

$\sigma_C = 0$ (since C is on neutral axis)

$\sigma_1 = \sigma_2 = 0$ *Ans.*

D-81. The beam is subjected to the loading shown. Determine the equation of the elastic curve. EI is constant.



Prob. D–81

D-81. $A_y = 3\text{ kN}$

Use section having a length x.

$\circlearrowleft + \Sigma M = 0; \quad -3x + 2x\left(\frac{x}{2}\right) + M = 0$

$M = 3x - x^2$

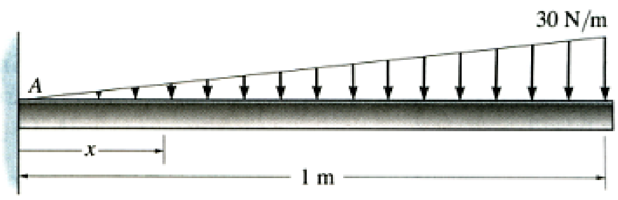
$EI\,\frac{d^2v}{dx^2} = 3x - x^2$

Integrate twice, use

$v = 0$ at $x = 0$, $\quad v = 0$ at $x = 3\text{ m}$

$v = \frac{1}{EI}\left(-\frac{1}{12}x^4 + 0.5x^3 - 2.25x\right)$

D-82. The beam is subjected to the loading shown. Determine the equation of the elastic curve. EI is constant.



Prob. D–82

D-82. $A_y = 15\text{ N}$

$M_A = 10\text{ N}\cdot\text{m}$

Use section having a length x.

Intensity of $w = 30x$ at x.

$\circlearrowleft + \Sigma M = 0;$

$-15x + 10 + \left(\frac{1}{3}x\right)\left[\frac{1}{2}(30x)(x)\right] + M = 0$

$M = 15x - 5x^3 - 10$

$EI\,\frac{d^2v}{dx^2} = 15x - 5x^3 - 10$

Integrate twice, use

$v = 0$ at $x = 0$,

$dv/dx = 0$ at $x = 0$

$v = \frac{1}{EI}\left(2.5x^3 - 0.25x^3 - 5x^2\right)$ *Ans.*

Buckling of Columns

D-85. The critical load is the maximum axial load that a column can support when it is on the verge of buckling. This loading represents a case of neutral equilibrium. True or false?

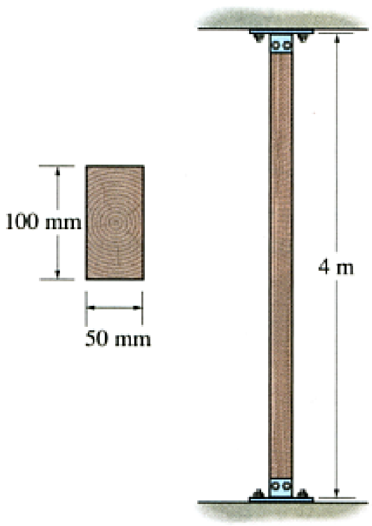
D-85. True. *Ans.*

D-86. A 1250-mm-long rod is made from a 25-mm-diameter steel rod. Determine the critical buckling load if the ends are fixed supported. $E = 210(10^3)$ MPa, $\sigma_Y = 260$ MPa.

D-86. $$P = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(210 \times 10^3)(\frac{\pi}{4}(12.5)^4)}{[0.5(12.50)]^2} = 101.7 \text{ kN} \quad \textit{Ans.}$$

$$\sigma = \frac{P}{A} = \frac{101.7 \times 10^3}{\pi(12.5)^2} = 207.2 \text{ MPa} < \sigma_Y \text{ OK}$$

D-87. A 4-m wooden rectangular column has the dimensions shown. Determine the critical load if the ends are assumed to be pin-connected. $E = 11 \times 10^3$ MPa. Yielding does not occur.



Prob. D–87

D-87. $$P = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(11)(10^3)[\frac{1}{12}(100)(50^3)]}{[1(4000)]^2} = 7.07 \text{ kN} \quad \textit{Ans.}$$

D-88. A steel pipe is fixed-supported at its ends. If it is 5 m long and has an outer diameter of 50 mm and a thickness of 10 mm, determine the maximum axial load P that it can carry without buckling. $E_{st} = 200$ GPa, $\sigma_Y = 250$ MPa.

D-88. $A = \pi((0.025)^2 - (0.015)^2) = 1.257(10^{-3})\ \text{m}^2$

$I = \frac{1}{4}\pi((0.025)^4 - (0.015)^4) = 267.04(10^{-9})\ \text{m}^4$

$$P = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2(200(10^9))(267.04)(10^{-9})}{[0.5(5)]^2}$$

$= 84.3$ kN *Ans.*

$$\sigma = \frac{P}{A} = \frac{84.3(10^3)}{1.257(10^{-3})} = 67.1\ \text{MPa} < 250\ \text{MPa OK}$$

D-89. A steel pipe is pin-supported at its ends. If it is 2 m long and has an outer diameter of 50 mm, determine its smallest thickness so that it can support an axial load of $P = 180$ kN without buckling. $E_{st} = 210(10^3)$ MPa, $\sigma_Y = 260$ MPa.

D-89. $P = \frac{\pi^2 EI}{(KL)^2};$

$$180(10^3) = \frac{\pi^2 210(10^3)[\frac{\pi}{4}(25^4 - r_2^4)]}{(1 \times 2000)^2}$$

$r_2 = 15.08$ mm

$$\sigma = \frac{P}{A} = \frac{180(10^3)}{\pi[(25)^2 - (15.08)^2]} = 144.1\ \text{MPa} < 260\ \text{MPa OK}$$

Thus, $t = 25 - 15.08 = 9.92$ mm *Ans.*

D-90. Determine the smallest diameter of a solid 1000-mm-long steel rod, to the nearest mm, that will support an axial load of $P = 15$ kN, without buckling. The ends are pin connected. $E_{st} = 210(10^3)$ MPa, $\sigma_Y = 260$ MPa.

D-90. $P = \frac{\pi^2 EI}{(KL)^2};$

$$15(10^3) = \frac{\pi^2 210(10^3)\frac{\pi}{4}r^4}{[1(1000)]^2}$$

$\therefore r_2 = 9.8$ mm

$$\sigma = \frac{P}{A} = \frac{15(10^3)}{\pi(9.8^2)} = 49.7\ \text{MPa} < 260\ \text{MPa OK}$$

$d = 2r = 19.6$ mm

Use $d = 20$ mm *Ans.*