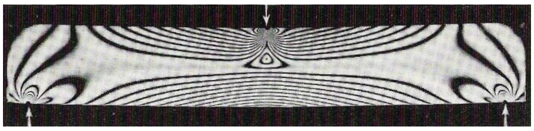
GRAĐEVINSKI FAKULTET SVEUČILIŠTA U RIJECI
OTPORNOST MATERIJALA



$$\sigma = F/A \qquad \varepsilon = \frac{(l+\Delta l)-l}{l} = \frac{\Delta l}{l} \qquad \Delta l = \alpha_T \cdot \Delta t \cdot l + \frac{F \cdot l}{E \cdot A} \qquad E = \frac{\sigma}{\varepsilon} \qquad \upsilon = \left|\varepsilon_p / \varepsilon_u\right|$$

$$[\sigma_{ij}] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} \qquad \vec{\rho}_n = [\sigma_{ij}] \cdot \vec{n} \qquad \begin{bmatrix} \rho_{nx} \\ \rho_{ny} \\ \rho_{nz} \end{bmatrix} = [\sigma_{ij}] \cdot \begin{bmatrix} \cos(x,n) \\ \cos(y,n) \\ \cos(z,n) \end{bmatrix} \qquad \rho_n = \sqrt{\rho_{nx}^2 + \rho_{ny}^2 + \rho_{nz}^2}$$

$$\sigma_n = \vec{\rho}_n \cdot \vec{n} \qquad \tau_n = \sqrt{\rho_n^2 - \sigma_n^2}$$

$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_{yy} & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_x - \sigma_s & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_s & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_z \end{bmatrix} + \begin{bmatrix} \sigma_s & 0 & 0 \\ 0 & \sigma_s & 0 \\ 0 & 0 & \sigma_s \end{bmatrix} \qquad \sigma_s = \tfrac{1}{3} \cdot (\sigma_x + \sigma_y + \sigma_z)$$

$$\sigma_{1,2} = \tfrac{1}{2} \cdot (\sigma_{xx} + \sigma_{yy}) \pm \tfrac{1}{2} \cdot \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4 \cdot \tau_{xy}^2} \qquad \mathrm{tg}\varphi_{oi} = \frac{\tau_{yx}}{\sigma_i - \sigma_{yy}} \qquad |\varphi_{01}| + |\varphi_{02}| = 90° \qquad \mathrm{tg}(2 \cdot \varphi_o) = \frac{2 \cdot \tau_{xy}}{\sigma_{xx} - \sigma_{yy}}$$

$$\sigma_{xx} + \sigma_{yy} + \sigma_{zz} = \sigma_1 + \sigma_2 + \sigma_3 \qquad \sigma_1 \geq \sigma_2 \geq \sigma_3 \qquad \varphi_1 = \varphi_0 + \frac{\pi}{4}$$

$$\sigma_{nn} = \sigma_{xx} \cdot \cos^2\varphi + \sigma_{yy} \cdot \sin^2\varphi + \tau_{xy} \cdot \sin(2\cdot\varphi) = \tfrac{1}{2} \cdot (\sigma_{xx} + \sigma_{yy}) + \tfrac{1}{2} \cdot (\sigma_{xx} - \sigma_{yy}) \cdot \cos(2\cdot\varphi) + \tau_{xy} \cdot \sin(2\cdot\varphi)$$

$$\tau_{nt} = \tfrac{1}{2} \cdot (\sigma_{yy} - \sigma_{xx}) \cdot \sin(2\cdot\varphi) + \tau_{xy} \cdot \cos(2\cdot\varphi)$$

$$\tau_{12} = \pm\tfrac{1}{2} \cdot \sqrt{(\sigma_{xx} - \sigma_{yy})^2 + 4 \cdot \tau_{xy}^2} = \pm 0.5 \cdot (\sigma_1 - \sigma_2) \qquad \tau_{MAX} = 0.5 \cdot (\sigma_1 - \sigma_2) \qquad \sigma(\varphi_1) = 0.5 \cdot (\sigma_1 + \sigma_2)$$

$$\sigma_{OKT} = \tfrac{1}{3} \cdot (\sigma_1 + \sigma_2 + \sigma_3) = \tfrac{1}{3} \cdot (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

$$\tau_{OKT} = \tfrac{1}{3} \cdot \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \qquad \rho_{OKT} = \sqrt{\sigma_{OKT}^2 + \tau_{OKT}^2}$$

$$\sigma_{xx} = \frac{E}{(1+\upsilon) \cdot (1 - 2 \cdot \upsilon)} \cdot [(1-\upsilon) \cdot \varepsilon_{xx} + \upsilon \cdot (\varepsilon_{yy} + \varepsilon_{zz})]$$

$$\varepsilon_{\substack{xx \\ yy \\ zz}} = \tfrac{1}{E}\left[\sigma_{\substack{xx \\ yy \\ zz}} - \upsilon \cdot (\sigma_{\substack{yy \\ zz \\ xx}} + \sigma_{\substack{zz \\ xx \\ yy}})\right] + \alpha_T \cdot \Delta t$$

$$\sigma_{\substack{xx \\ yy \\ zz}} = \frac{E}{1-\upsilon^2} \cdot \left[\varepsilon_{\substack{xx \\ yy \\ zz}} + \upsilon \cdot \varepsilon_{\substack{yy \\ xx \\ yy}}\right]$$

$$\varepsilon_{nn} = \varepsilon_{\substack{xx \\ yy}} \cdot \cos^2\varphi + \varepsilon_{\substack{yy \\ zz}} \cdot \sin^2\varphi + \varepsilon_{\substack{xy \\ yz}} \cdot \sin(2\cdot\varphi)$$

$$\varepsilon_{xy} = \frac{\tau_{xy}}{2 \cdot G} = \frac{1+\upsilon}{E} \cdot \tau_{xy} = \frac{\gamma_{xy}}{2} \qquad \varepsilon_{\substack{xy \\ yz \\ zx}} = \frac{1}{2 \cdot G} \cdot \tau_{\substack{xy \\ yz \\ zx}}$$

$$\varepsilon_d = (\varepsilon_x \cdot \cos^2\alpha + \varepsilon_y \cdot \cos^2\beta + \varepsilon_z \cdot \cos^2\gamma) \qquad G = \frac{E}{2 \cdot (1+\upsilon)}$$

$$\varepsilon_V = (\varepsilon_1 + \varepsilon_2 + \varepsilon_3) = \tfrac{1}{E} \cdot (1 - 2 \cdot \upsilon) \cdot (\sigma_1 + \sigma_2 + \sigma_3) + (3 \cdot \alpha_T \cdot \Delta t) = (\varepsilon_x + \varepsilon_y + \varepsilon_z) = \Delta V / V$$

ZAKOVICE $F_H = \dfrac{H}{n}$ $\quad F_V = \dfrac{F}{n}$ $\quad F_{ni} = \dfrac{M}{\sum \rho_i^2} \cdot \rho_i$ $\quad R = \sqrt{F_x^2 + F_y^2}$ $\quad \tau = \dfrac{R}{x \cdot \dfrac{d^2 \cdot \Pi}{4}} \leq \tau_{DOP}$ $\quad \sigma = \dfrac{R}{d \cdot t} \leq \sigma_{DOP}$

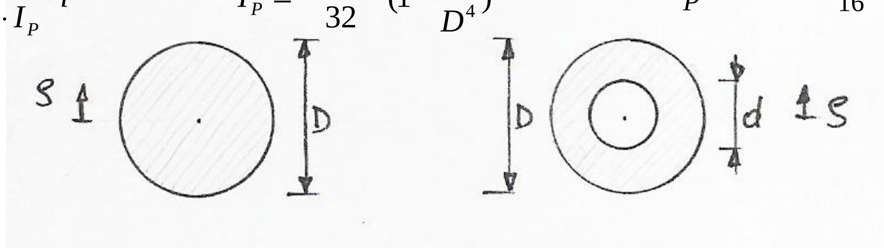
n = BROJ ZAKOVICA $\qquad$ *X*= REZNOST ZAKOVICA

ZAVAR $\sigma = \dfrac{F}{b \cdot t}$ $\quad \tau = \dfrac{F}{x \cdot a \cdot l} \leq \tau_{DOP}$ $\quad a = 0.7 \cdot t$

x=BROJ VAROVA

TORZIJA $\tau = \dfrac{M_t}{I_p} \cdot \rho$ $\quad \tau_{MAX} = \dfrac{M_T}{I_p} \cdot r = \dfrac{M_t}{W_p}$ $\quad I_P = \dfrac{\Pi \cdot D^4}{32}$ $\quad W_p = \dfrac{I_p}{r}$ $\quad W_p = \dfrac{\Pi \cdot D^3}{16}$

$$\varphi = \frac{M_t}{G \cdot I_P} \cdot l \qquad I_P = \frac{\Pi \cdot D^4}{32}\left(1 - \frac{d^4}{D^4}\right) \qquad W_P = \frac{\Pi \cdot D^3}{16} \cdot \left(1 - \frac{d^4}{D^4}\right)$$



POSUDE TANKIH STIJENKI $\varepsilon_1 = \frac{1}{E} \cdot (\sigma_1 - \upsilon \cdot \sigma_2)$ $\frac{\sigma_1}{\rho_1} + \frac{\sigma_2}{\rho_2} = \frac{p}{h}$

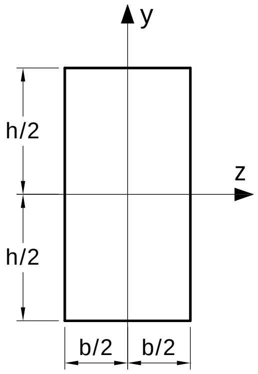
KUGLA $\sigma_1 = \sigma_2 = \sigma$ $\rho_1 = \rho_2 = r$ $\sigma = \frac{p \cdot r}{2 \cdot h}$ $\varepsilon = \frac{\Delta r}{r} = \frac{1-\upsilon}{E} \cdot \sigma$ $\Delta r = \frac{1-\upsilon}{E} \cdot \frac{p \cdot r^2}{2 \cdot h}$

CILINDRIČNA POSODA $\rho_1 = r$ $\rho_2 = \infty$ $\sigma_1 = \frac{p \cdot r}{h}$ $\sigma_2 = \frac{p \cdot r}{2 \cdot h}$ $\varepsilon_1 = \frac{1 - 0.5 \cdot \upsilon}{E} \cdot \sigma_1 = \frac{\Delta r}{r}$

PRSTEN $\rho_1 = r$ $\rho_2 = \infty$ $\sigma_1 = \sigma$ $\sigma_2 = 0$ $\sigma = \frac{p \cdot r}{h}$ $\varepsilon = \frac{\Delta r}{r} = \frac{\Delta \mathrm{D}}{\mathrm{D}}$ $\Delta r = \frac{p \cdot r^2}{E \cdot h}$

GEOMETRIJSKE KARAKTERISTIKE PRAVOKUTNIKA

$I_z = \frac{b \cdot h^3}{12}$ $I_y = \frac{h \cdot b^3}{12}$ $W_z = \frac{bh^2}{6}$ $W_y = \frac{hb^2}{6}$ $i_z = \sqrt{\frac{I_z}{A}}$ $i_y = \sqrt{\frac{I_y}{A}}$



STEINEROV TEOREM $I_Y = I_{y,VL} + d^2 \cdot A$ (d – UDALJENOST OD OSI y DO TEŽIŠTA A)

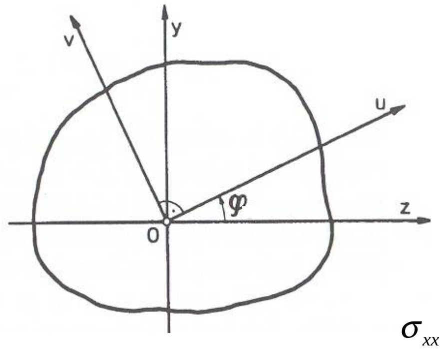
GLAVNE OSI PRESJEKA

$$I_{1,2} = \tfrac{1}{2} \cdot (I_z + I_y) \pm \tfrac{1}{2} \cdot \sqrt{(I_z - I_y)^2 + 4 \cdot I_{zy}^2}$$

$$I_z > I_y \rightarrow I_U = I_{MAX} \rightarrow I_V = I_{MIN}$$

$$I_z < I_y \rightarrow I_U = I_{MIN} \rightarrow I_V = I_{MAX}$$

$$\mathrm{tg}(2 \cdot \varphi) = -2 \cdot \frac{I_{zy}}{I_z - I_y} \qquad I_1 + I_2 = I_z + I_y$$



SAVIJANJE $\sigma_{xxMAX} = \frac{M_{MAX}}{I_y} \cdot z_{MAX} = \frac{M}{W_y}$ $\tau_{xz} = \frac{T_z \cdot S_y}{I_y \cdot b}$ $\sigma_{xx} = \frac{M}{I_y} \cdot z \pm \frac{N}{A} < \sigma_{DOP}$

KOSO SAVIJANJE $M_y = M \cdot \cos\alpha$ $M_z = M \cdot \sin\alpha$ $\sigma_{xx} = \pm(\frac{M_y}{I_y} \cdot z + \frac{M_z}{I_z} \cdot y) \leq \sigma_{DOP}$ $\mathrm{tg}\varphi = -\frac{I_y}{I_z} \cdot \mathrm{tg}\alpha$

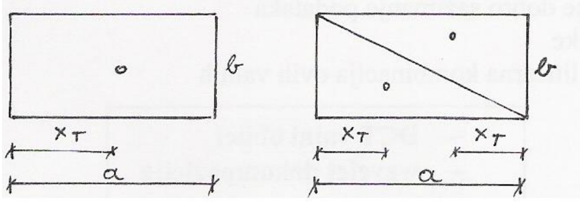
GRAFOANALITIČKI

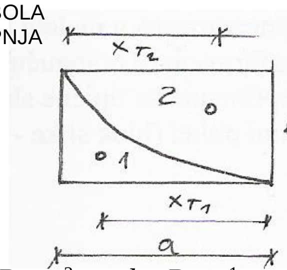
PARABOLA 2 STUPNJA

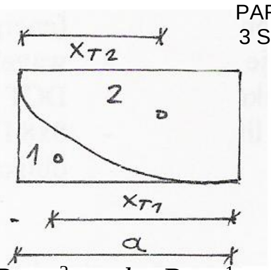
PARABOLA 3 STUPNJA

$\bar{q} = \frac{M}{E \cdot I}$

$\varphi = \frac{\bar{T}}{E \cdot I}$







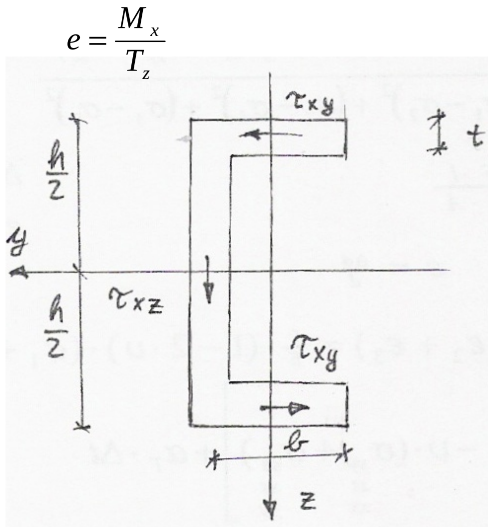
$w = \frac{\bar{M}}{E \cdot I}$ $P = a \cdot b$ $P = 0.5 \cdot a \cdot b$ $P_2 = \frac{2}{3} \cdot a \cdot b$ $P_1 = \frac{1}{3} \cdot a \cdot b$ $P_2 = \frac{3}{4} \cdot a \cdot b$ $P_1 = \frac{1}{4} \cdot a \cdot b$

$x_T = 0.5 \cdot a$ $x_T = \frac{1}{3} \cdot a$ $x_{T2} = \frac{5}{8} \cdot a$ $x_{T1} = \frac{3}{4} \cdot a$ $x_{T2} = \frac{3}{5} \cdot a$ $x_{T1} = \frac{4}{5} \cdot a$

ANALITIČKI $-q(x) = \frac{dT(x)}{dx} = \frac{d^2M(x)}{dx^2}$ $\frac{-M(x)}{EI} = \frac{d\varphi}{dx}$ $\frac{-M(x)}{EI} = \frac{d^2w}{dx^2}$ $(x - a_i) > 0$

CENTAR TORZIJE

$M_x = \int_A (\tau_{xz} \cdot y - \tau_{xy} \cdot z) dA$ $T_z \cdot e = M_x = \int_A (\tau_{xz} \cdot y - \tau_{xy} \cdot z) dA$ $e = \frac{M_x}{T_z}$



$\tau = \frac{T \cdot S}{I \cdot b}$ $\tau_{xy} = \frac{T}{I_y \cdot t} \cdot (t \cdot S \cdot 0.5 \cdot h)$

$$\tau_{xz} = \frac{T}{I_y \cdot t} \cdot \left[0.5 \cdot t \cdot b \cdot h + 0.5 \cdot t \cdot (0.25 \cdot h^2 - z^2)\right]$$

$T_1 = \int_0^b \tau_{xy} \cdot t \, ds = 0.5 \cdot \tau_{xy} \cdot b \cdot t$ $T_2 = \int_{\frac{-h}{2}}^{\frac{h}{2}} \tau_{xz} \cdot t \, dz$

$T_z \cdot e = T_1 \cdot h$

MOŽDANICI $\sigma_x = \frac{M_{MAX}}{f \cdot W_{yn}}$ f=PODATLJIVOST SPOJA $R_x = \frac{T_{Z(MAX)} \cdot S_{yb}}{I_{yb}} \cdot l$ R_x=HORIZONTALNA POSMIČNA SILA NA DUŽINI *l*

$$\sigma_o = \frac{R_x}{c \cdot b} \le \sigma_{DOP} \qquad \tau = \frac{R_x}{a \cdot b} \le \tau_{DOP} \qquad \tau = \frac{R_x}{b \cdot (l-a)} \le \tau_{DOP}$$

TEORIJE ČVRSTOĆE $\sigma_E < \sigma_{DOP}$ $\qquad \sigma_D = \frac{\sigma_E}{K}$ $\qquad \sigma_E = \frac{M_E}{W_y} < \sigma_D$

1.NAJVEĆA NORMALNA

a)TROOSNO $\sigma_E = |\sigma_1| \le \sigma_{DOP}$

b)SAVIJANJE $\sigma_E = 0.5 \cdot \sigma + 0.5 \cdot \sqrt{\sigma^2 + 4 \cdot \tau^2}$

c)SAVIJANJE I TORZIJA $M_E = 0.5 \cdot (M_S + \sqrt{M_S^2 + M_T^2})$

2.NORMALNE DEFORMACIJE

a) $\sigma_E = \sigma_1 - \upsilon \cdot (\sigma_2 + \sigma_3) \le \sigma_{DOP}$

b) $\sigma_E = 0.5 \cdot (1-\upsilon) \cdot \sigma + 0.5 \cdot (1+\upsilon) \cdot \sqrt{\sigma^2 + 4 \cdot \tau^2}$

c) $M_E = 0.5 \cdot (1-\upsilon) \cdot M_S + 0.5 \cdot (1+\upsilon) \cdot \sqrt{M_S^2 + M_T^2}$

3.POSMIČNA NAPREZANJA

a) $\sigma_E = \sigma_1 - \sigma_3 \le \sigma_{DOP}$

b) $\sigma_E = \sqrt{\sigma^2 + 4 \cdot \tau^2}$

c) $M_E = \sqrt{M_S^2 + M_T^2}$

4.POTENCIJALNA ENERGIJA DEFORMACIJA

a) $\sigma_E = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2 \cdot \upsilon \cdot (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)} \le \sigma_{DOP}$

b) $\sigma_E = \sqrt{\sigma^2 + 2 \cdot (1+\upsilon) \cdot \tau^2}$

c) $M_E = \sqrt{M_S^2 + 0.5 \cdot (1+\upsilon) \cdot M_T^2}$

5. POTENCIJALNA ENERGIJA PROMJENE OBLIKA

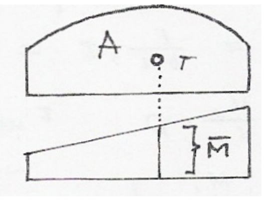
a) $\sigma_E = \sqrt{0.5 \cdot [\ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\ ]} \le \sigma_{DOP}$

b) $\sigma_E = \sqrt{\sigma^2 + 3 \cdot \tau^2}$

c) $M_E = \sqrt{M_S^2 + 0.75 \cdot M_T^2}$

ENERGIJA $U = \int_0^l \frac{N^2 dx}{2 \cdot E \cdot A} + \int_0^l k_y \cdot \frac{T_y^2 dx}{2 \cdot G \cdot A} + \int_0^l k_z \cdot \frac{T_z^2 dx}{2 \cdot G \cdot A} + \int_0^l \frac{M_T^2 dx}{2 \cdot G \cdot I_T} + \int_0^l \frac{M_y^2 dx}{2 \cdot E \cdot I_y} + \int_0^l \frac{M_z^2 dx}{2 \cdot E \cdot I_z}$

$$\delta = \frac{\partial U}{\partial F} \qquad \varphi = \frac{\partial U}{\partial M} \qquad F = \frac{\partial U}{\partial \delta}$$



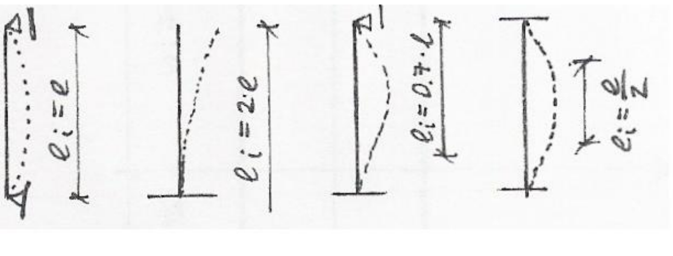
METODA JEDINIČNOG OPTEREČENJA $\delta_k = \int_0^l \frac{N \cdot \overline{N}_k dx}{E \cdot A} + ... + \int_0^l \frac{M_y \cdot \overline{M}_{yk} dx}{E \cdot I_y}$

$$\delta_k = \sum_{i=1}^{n} \frac{A_{Ni} \cdot \overline{N}_{ki}}{(E \cdot A)_i} + ... + \sum_{i=1}^{n} \frac{A_{Mi} \cdot \overline{M}_{ki}}{(E \cdot I)_i}$$

IZVIJANJE

$$F_{KR} = \frac{\Pi^2 \cdot EI_{MIN}}{l_i^2} \qquad \sigma_{KR} = \frac{F_{KR}}{A} = \frac{\Pi^2 \cdot EI_{MIN}}{A \cdot l_i^2} \qquad i_{MIN} = \sqrt{\frac{I_{MIN}}{A}} \qquad \lambda = \frac{l_i}{i_{MIN}} \qquad \lambda^2 = \frac{l_i^2}{i_{MIN}^2} = \frac{A \cdot l_i^2}{I_{MIN}}$$

$$\sigma_{KR} = \frac{\Pi^2 \cdot E}{\lambda^2} \le \sigma_p \qquad \lambda_p = \sqrt{\frac{\Pi^2 \cdot E}{\sigma_p}} \qquad \sigma_{DOP} = \frac{\sigma_{KR}}{k}$$



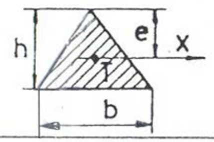
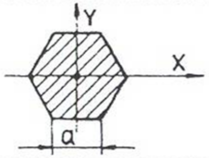
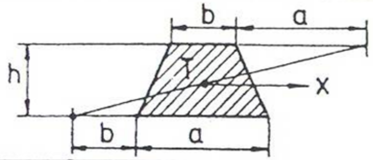
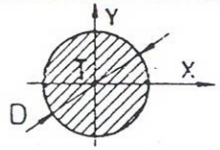
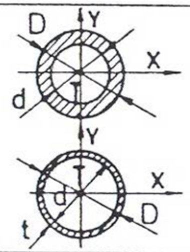
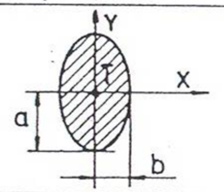
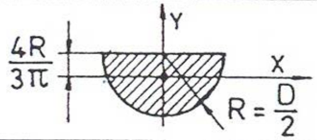
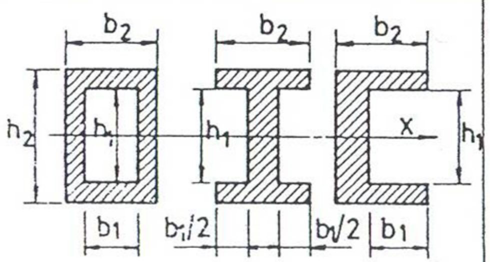
EULEROVA HIPERBOLA $\sigma_{KR} = \frac{\Pi^2 \cdot E}{\lambda^2}$

TETMAYEROV PRAVAC $\sigma_{KR} = a - b \cdot \lambda$

TEORIJA PLASTIČNOSTI $W_{pl} = S_T + S_V \qquad S_T = A_T \cdot h_T \qquad S_V = A_V \cdot h_V \qquad A_T = A_V$

$$W_V + W_U = 0 \qquad W_i = F \cdot \delta + M \cdot \varphi$$

$$W_{pl} = \frac{b \cdot h^2}{4} \qquad M_{pl} = \sigma_T \cdot W_{pl}$$

 $e = \frac{2}{3}h$	$I_x = h^3 b/36$	$W_x = h^2 b/24$
	$I_x = I_y = 0.5413\, a^4$	$W_x = 0,625 a^3$ $W_y = 0,5413\, a^3$
	$I_x = \frac{h^3}{36} \frac{a^2 + 4ab + b^2}{a+b}$	$W_x = \frac{h^2}{12} \frac{a^2 + 4ab + b^2}{2a+b}$
	$I_x = I_y = \frac{\pi D^4}{64}$	$W_x = W_y = \frac{\pi D^3}{32}$
a) b) 	a) $I_x = I_y = \frac{\pi}{64}(D^4 - d^4)$ za malu debljinu stijenke t : b) $I_x = I_y \cong \pi R_{sr}^3 t$	$W_x = W_y = \frac{\pi}{32} \cdot \frac{(D^4 - d^4)}{D}$ $W_x = W_y \cong R_{sr}^2 t$ $R_{sr} = \frac{(D+d)}{2}$
	$I_x = \pi a^3 b/4$ $I_y = \pi b^3 a/4$	$W_x = \pi a^2 b/4$ $W_y = \pi b^2 a/4$
	$I_x \cong 0,007 D^4$	$W_x \cong 0,024 D^3$
	$I_x = \frac{b_2 h_2^3 - b_1 h_1^3}{12}$	$W_x = \frac{b_2 h_2^3 - b_1 h_1^3}{6 h_2}$