RAZLOMCI

zbrajanje	množenje	dijeljenje	dvostruki
$\frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd}$	$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} = \frac{ac}{bd}$	$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \times d}{b \times c}$

- pravilo asocijacije: $(r_1 + r_2) + r_3 = r_1 + (r_2 + r_3)$

REALNI BROJEVI

zbrajanje	množenje
$(x + y) + z = x + (y + z)$	$(x \times y) \times z = x \times (y \times z)$

KVADRAT I KUB BINOMA

osnovna	kvadratne razlike	razlika kvadrata	kub binoma
$(a+b)^2 = a^2 + 2ab + b^2$	$(a-b)^2 = a^2 - 2ab + b^2$	$(a^2-b^2) = (a-b)(a+b)$	$(a \overset{+}{-} b)^3 = a^3 \overset{+}{-} 3a^2b \overset{+}{-} 3ab^2 \overset{-}{+} b^3$

RAZLIKA I ZBROJ KUBOVA

zbrajanje	oduzimanje
$a^3 + b^3 = (a+b)(a^2 + ab + b^2)$	$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

APSOLUTNE VRIJEDNOSTI

množenje	dijeljenje	zbrajanje
$\lfloor a \times b \rfloor = \lfloor a \rfloor \times \lfloor b \rfloor$	$\left\lfloor \frac{a}{b} \right\rfloor = \frac{\lfloor a \rfloor}{\lfloor b \rfloor}$	$\lfloor a + b \rfloor \leq \lfloor a \rfloor + \lfloor b \rfloor$

POTENCIJE

općenito	zbrajanje i oduzimanje	množenje i dijeljenje	množenje i dijeljenje potenc. jednakih eksponenata
$a^1 = a$ $a^0 = 1$ $a^{-n} = \frac{1}{n}$ $(-1)^{2n} = 1$ $(-1)^{3n} = -1$ $(-3n = 2n+1)$	samo ako imaju jednake BAZE i eksponente, a zbroje i oduzmu se tako da se koef. zbroje, oduzmu a baze prepišu	$a^m \times a^n = a^{m+n}$ $a^m : a^n = a^{m-n}$	$a^n \times b^n = (a \times b)^n$ $a^n : b^n = (a : b)^n$

potenciranje potencija: $(a^m)^n = a^{m\,n}$

KORIJENI

$\sqrt[n]{a} = b$ n – eksponent; a- radikant; b – korijen

$b^n = a$

$\sqrt{x^2} = |x|$

skraćivanje i proširivanje	unošenje pod znak korijena	djelomično korijenovanje
$\sqrt[n]{a^m} = \sqrt[mr]{a^{mr}}$	$a\sqrt[n]{b} = \sqrt[n]{a^n \times b}$	$\sqrt[n]{a^n \times b} = a\sqrt[n]{b}$
zbrajanje i oduzimanje	**množenje i dijeljenje**	**potenciranje korijena**
oni koji imaju jednake eksponente i radikante $3\sqrt{2} + 4\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}$	$\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{a \times b}$ $\sqrt[n]{a} \times \sqrt[m]{b} = \sqrt[mn]{a^m \times b^n}$ $\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{a : b}$	$\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^m}$

korjenovanje korijena	potencija sa razlomljenim eksponentom	racionalizacija nazivnika
$\sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$	$a = \sqrt[n]{a^m}$	$\frac{1}{a} \times \frac{a}{a} = \frac{a}{a}$ $\frac{1}{a - b} - \frac{b + b}{a + b} = \frac{a + b}{a - b}$

IRACIONALNE JEDNADŽBE

$\sqrt{a-x} = b\sqrt{a+c} \quad /^2$

$a - x = b(x+c)$ a, b i c su bilo koji broj

SKUP KOMPLEKSNIH BROJEVA

$x^2 + 1 = 0$; $x^2 = -1$; $x = +- \sqrt{-1}$; $2= a + bi$; $2= x + xi$

JEDNADŽBE

$ax = b$; $ax = b/:a$; $x = \frac{b}{a}$

$\frac{ax + b}{ax - b} = c \quad / \text{nazivnik}_{ax - b}$

$ax+b = c(ax-b)$ a, b i c su bilo koji broj

KVADRATKNE JEDNADŽBE

$ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b +- \sqrt{b^2 - 4ac}}{2a}$$

BIKVDRATNE JEDNADŽBE

$ax^4 + bx^2 + c = 0$
$t = x^2$

$at^2 + bt + c = 0$

$$t_{1,2} = \frac{-b +- \sqrt{b^2 - 4ac}}{2a}$$

OMJERI I RAZMJERI

$a \times d = c \times b$

LINEARNA (AFINA) FUNKCIJA

$f(x) = ax + b$

kvadratna funkcija

$f(x) = ax^2 + bx + c$

tjeme

$$T\left(-\frac{b}{a}, \frac{b^2 - 4ac}{4a} \text{ ili } \frac{4ac - b^2}{4a}\right)$$

nul točke

$ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b +- \sqrt{b^2 - 4ac}}{2a}$$

SUSTAV DVIJE LINEARNE JEDN. SA DVIJE NEPOZNANICE

a) $a_1x + b_1y + c_1 = 0$
b) eliminacija x ili y
c) komparacija

$ax + by - c = 0$
$ax - y - c = 0 \Rightarrow -y = -ax + c \;/(-1)$
$y = ax - c$
$ax + by - c = o$
$ax + bx - c - c = 0$
x = neki broj, a y= izračuna se

NEJEDNADŽBE (prebacivanje na drugu stranu i $x_{1,2}$)

$ax + b > 0$

$ax > b/:a$

$x > -\frac{b}{a}$

SUSTAV LIN. JEDN. S JEDNOM NEPOZNANICOM (graf)

$ax + b \geq 0$

$cx + d \geq 0$

$ax + b \geq cx - d$

$fx + g > hx - k$

NEJEDNADŽBE S ALGEBARSKIM RAZLOMCIMA (tablice od -∞ do +∞)

$\frac{ax + b}{cx + d}$

$\frac{x + a}{x - b} < -c$

ili

$\frac{ax + b}{ax + d}$ $\leq$ $\geq$ = ● $<$ $>$ = ○

1. Izračunajte

a) $\frac{1}{2} \times (\frac{1}{2} - \frac{1}{5}) - \frac{1}{5} \times (\frac{1}{2} - \frac{1}{5}) = (\frac{1}{2} \times \frac{3}{10}) - (\frac{1}{5} \times \frac{3}{10}) = \frac{3}{20} - \frac{3}{50} = \frac{15-6}{100} = \mathbf{\frac{9}{100}}$

a) $10 : (\frac{4}{5}) = 10 : \frac{23}{50} = \frac{40}{23} = \frac{\cancel{10}}{1} \times \frac{23}{\cancel{40}_4} = \mathbf{\frac{23}{4}}$

b) $\frac{\frac{2}{5} - \frac{1}{7}}{\frac{2}{5} + \frac{1}{7}} \times (\frac{7}{2} - \frac{1}{3}) = \frac{\frac{9}{35}}{\frac{19}{35}} \times \frac{19}{6} = \frac{9}{6} = \mathbf{\frac{3}{2}}$

c) $(\frac{1}{2} : \frac{5}{4} + \frac{7}{5} : \frac{11}{7} - \frac{3}{11}) : (\frac{3}{2} + \frac{1}{4}) : \frac{55}{3} = (\frac{1}{2} \times \frac{4}{5} + \frac{7}{5} \times \frac{7}{11} - \frac{3}{11}) : (\frac{7}{4} \times \frac{3}{55}) = \frac{56}{55} : \frac{21}{220} = \frac{56}{55} \times \frac{220}{21} = \mathbf{\frac{32}{3}}$

2. Pojednostavnite:

a) $\frac{x^2}{x^2 - xy} + \frac{y^2}{xy - y^2} - \frac{2y}{x-y} = \frac{x^2}{x(x-y)} + \frac{y^2}{y(x-y)} - \frac{2y}{x-y} = \frac{x^2y + y^2x - 2y^2x}{xy(x-y)} = \frac{\cancel{xy}\,(x+y-2y)}{\cancel{xy}\,(x-y)}$

b) $(\frac{x}{x+1} - \frac{x^2}{x^2+2x+1}) : (\frac{x}{x^2-1} - \frac{1}{x+1}) \times \frac{x+1}{x} = (\frac{x}{x+1} - \frac{x^2}{(x+1)^2}) : (\frac{x}{(x-1)(x+1)} - \frac{1}{x+1}) \times \frac{x+1}{x} =$

$= (\frac{x(x+1) - x^2}{(x+1)^2}) : (\frac{x - (x-1)}{(x-1)(x+1)}) \times \frac{x+1}{x} = \frac{x^2 + x - x^2}{(x+1)^2} : (\frac{x - x + 1}{(x-1)(x+1)}) \times \frac{x+1}{x} =$

$= \frac{x}{(x+1)^{\cancel{2}}} \times \frac{(x-1)\cancel{(x+1)}}{1} \times \frac{x+1}{x} = \mathbf{x-1}$

b) $\sqrt{a^3b^7} : \sqrt{ab^5} = \sqrt[2]{a^2\, a\, b^2\, b^2\, b^2\, b} : \sqrt{a\, b^2\, b^2\, b}$

$\frac{a\, \cancel{3b}^{1b}\, \cancel{\sqrt{ab}}}{\cancel{2b}\, \cancel{\sqrt{ab}}} = \mathbf{ab}$

3. Riješite slijedeće jednadžbe:

a) kvadratna (9x – 3) (6x – 7) = (18x – 1) (3x+ 2) – 1 = svaki sa svakim pomnožiti

a) $\mathbf{7^{3-5x} = 1}$

$7^{3-5x} = 7^0$

$3 - 5x = 0$

$-5x = -3 / (-5)$

$\mathbf{x = \frac{3}{5}}$

b) **kvadrat binoma** $\frac{1}{x^2 - 4x + 4} - \frac{2}{x^2 - 4} = \frac{3}{x^2 + 4x + 4}$ $\Big/ (x+2)^2 (x-2)^2$

$(x-2)^2 \quad (x-2)(x+2) \quad (x+2)^2$

$(x+2)^2 - 2(x-2)(x+2) = 3(x-2)^2$

$x^2 + 4x + 4 - 2(x^2 - 4) = 3(x^2 - 4x - 4)$

$x^2 + 4x + 4 - 2x^2 + 8 = 3x^2 - 12x - 12$

$1 - 2x = 0;\ -2x = -1 / (-1);\ 2x = \frac{1}{2}$

$x_{1,2} =$

b) $x = \sqrt{12 - x} \quad / \; ^2$

$x^2 = 12 - x$

$1x^2 + 1x - 12 = 0 \qquad 2x^2 + 4x - 12 = 0$

$$x_{1,2} = \frac{-1 \pm \sqrt{1 + 4x12}}{2} = \frac{-1 \pm \sqrt{1 - 4x\,1\,x\,(-12)}}{2} = \frac{-1 \pm \sqrt{1 + 48}}{2} = \frac{1 \pm \sqrt{49}}{2} = \frac{1 \pm 7}{2}$$

$x_1 = \frac{\cancel{8}^{2}}{\cancel{2}} = \mathbf{4} \qquad x_2 = \frac{1 - 7}{2} = \frac{\cancel{-6}^{3}}{\cancel{2}} = \mathbf{-3}$

d) $\mathbf{1x^4 + 3x^2 - 4 = 0}$

$t = x^2$

$1t^2 + 3t - 4 = 0$

$$t_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4x1x(-4)}}{2} = \frac{-3 \pm \sqrt{9 + 16}}{2} = \frac{-3 \pm \sqrt{1 + 48}}{2} = \frac{1 \pm \sqrt{25}}{2} = \frac{-3 \pm 5}{2}$$

$t_1 = \frac{-3 + 5}{2} = \mathbf{1} \qquad t_2 = \frac{-3 - 5}{2} = \mathbf{-4}$

$t = x^2$	$t = x^2$	$t = x^2$
$x^2 = 1$	$x^2 = 1$	$x^2 = -4$
$x = \pm\sqrt{1}$	$x = \pm\sqrt{1}$	$x = \sqrt{-4}$
$x_2 = -\sqrt{1}$	$x_2 = \pm 3$	$x = \pm 2i$

4. Riješite nejednadžbe:

a) $\mathbf{\frac{5x - 2}{8} > \frac{x}{2} - \frac{2 - x}{6}}$

$\frac{5x - 2}{8} - \frac{x}{2} + \frac{2 - x}{6} > 0 \quad / \; 24$

$15x - 6 - 12x + 8 - 4x > 0$

$-x + 2 > 0 \qquad$ **x E <- ∞, 2>**

$-x > -2 \;/\; (-1)$

$x < 2$

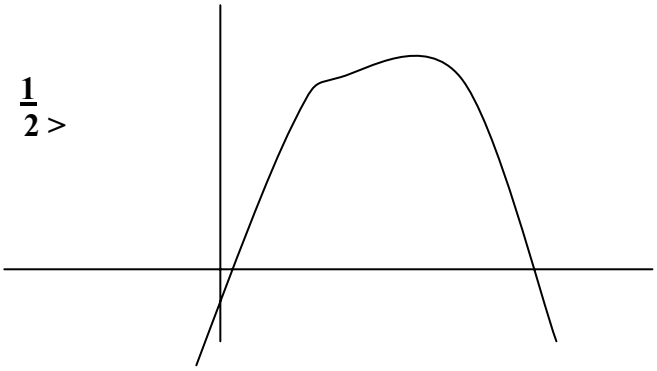
b) $\mathbf{-2x^2 + 7x - 3 \geq 0}$

$\mathbf{-2x^2 + 7x - 3 = 0}$

$$x_{1,2} = \frac{-7 \pm \sqrt{7^2 - 4x(-2x(-3)}}{2\,x\,(-2)} = \frac{-7 \pm \sqrt{49 + 24}}{-4} = \frac{-7 \pm \sqrt{25}}{2} = \frac{1 \pm \sqrt{25}}{-4} = \frac{-7 \pm 5}{-4}$$

$\mathbf{x_1 = \frac{1}{2}} \qquad \mathbf{x_2 = 3}$

N_1 ($\frac{1}{2}$, 0) **N_2 (3,0)** **x E [$\frac{1}{2}$, 3] U <-∞, $\frac{1}{2}$ >**



b) $\frac{x+2}{x-3} \le 2$

$\frac{x+2}{x-3} - 2 \le 0$

$\frac{x+2-2x+6}{x-3} \le 0$

$\frac{-1x+8}{x-3} \le 0$

	$-\infty$		3		8		$+\infty$
$-x+8$		+		+	●	-	
$x-3$		-	○	+		+	
rješenje		-		+		-	

$-x+8=0$
$-x=-8\ /(-1)$
$x=8$

$x-3=0$
$x=3$

rješenje: x E <- ∞, 3> U [8, + ∞>

algebarski razlomci

$\frac{12x^2y-18xy}{12x^2y-8x^2} = \frac{6xy\ (2x-3)}{4x^2(3xy-2x^2)} \rightarrow -4x^2(2x-3y)$

$\frac{\cancel{6}^{3}\ \cancel{x}y\ \cancel{(2x-3)}}{-4^{\cancel{2}}\ \cancel{x}\ \cancel{(2x-3y)}} = \frac{3y}{-2}$

$\frac{x^2-9}{(x+3)^2} = \frac{(x-3)\cancel{(x+3)}}{(x+3)^{\cancel{2}}} = \frac{(x-3)}{(x-3)}$

$\frac{3a+3b}{2a+2b} = \frac{3\cancel{(a+b)}}{2\cancel{(a+b)}} = \frac{3}{2}$

$\frac{a^2-1}{a^3-1} = \frac{(a-1)\ (a+1)}{(a-1)(a^2+a+1)} = \frac{a+1}{a^2+a+1}$

$\frac{4+a}{a+3} - \frac{a^2}{a^2+6a+9} = \frac{(4+a)(a+3)-a^2}{(a+3)^2} = \frac{4a+12+a^2+3a-a^2}{(a+3)^2} = \frac{3}{(x+3)^2}$

$\frac{x}{x^2-xy} + \frac{y}{xy-y^2} - \frac{2}{x-y} = \frac{xy+xy-2xy}{xy(x-y)} = \frac{\cancel{2xy}-\cancel{2xy}}{xy(x-y)} = 0$

potencije

$3ab^3 \times (-4a^2b^2) = -12a^3b^5$

$\frac{a^3b^8}{3a^2b^2} : \frac{ab^5}{6} = \frac{a^3}{3a^2b^2}\ \frac{b^8}{}\ \times \frac{b^2}{a^2b^5} = \frac{2a^3b^8}{a^2b^7} = \frac{2b^8}{b^7} = 2b$

$\frac{25}{16z} \times \frac{7y^{14}}{} \times \frac{14}{18x^8y^6}\ z = \frac{35x^7y^{14}z^8}{24z^4x^8y^6} = \frac{35}{24}x^{-1}y^8z^4$

korijeni

$\sqrt[3]{x^4\sqrt{x^3\sqrt[5]{x^2}}} = \sqrt[12]{x^4x^3\sqrt[5]{x^2}} = \sqrt[60]{x^{20}x^5x^2} = \sqrt[60]{x^{37}}$

$\sqrt{x^2+x+2} = x\ /^2$

$\cancel{x^2}+x-2=\cancel{x^2}$; $x=2$

iracionalne jednadžbe

$\sqrt{3-x} = 2\sqrt{x+4}\ /^2$
$3-x=4(x+4)$
$-5x=13$
$x=\frac{13}{5}$

provjera: $\sqrt{3+\frac{13}{15}} = 2\sqrt{-\frac{13}{15}+4}$

$\sqrt{\frac{28}{5}} = 2\sqrt{\frac{7}{5}}$

$3x-11=5(x+7)$
$3x-11=5x+35$
$3x-5x=35+11$
$-2x=46\ /(-2)$
$x=-23$

$5(2x+12)=3(x-2)+1$
$10x+60=3x-6+1$
$10x-3x=-6+1-60$
$7x=-65$
$x=-\frac{65}{7}$

$\frac{2}{3}(x-7) + \frac{1}{2} = \frac{1}{6}x$

$\frac{2}{3}x - \frac{14}{3} + \frac{1}{2} = \frac{1}{6}x/6$

$4x-28+3= x$
$4x-x = 28-3$
$3x= 25/:3$
$x= \frac{25}{3}$

$\frac{x-2}{4} - \frac{2x-3}{3} = 1$

$\frac{3x-6-(8x-12)}{12} = 1/12$

$3x-6-8x-12=12$
$3x-8x= 12+6+12$
$-5x-6=0$
$x= -\frac{6}{5}$

$\frac{2x-1}{3} - \frac{3x-1}{2} = \frac{-x-2}{2}$

$4x-2-(9x-3) = -3x-6$
$4x-2-9x+3=-3x-6$
$-2x+7=0$
$-2x=-7$
$x= \frac{7}{2}$

$\frac{3x+8}{2x-1} = 3/ 2x-1$

$3x+8= 3(2x-1)$
$3x+8= 6x-3$
$3x-6x= -8-3$
$-3x = 11/(-3)$
$x= \frac{11}{3}$

$\frac{8x+2}{4x-7} = 1/4x-7$

$8x+2= 4x-7$
$4x= 9/:4$
$x= -\frac{9}{4}$

$\frac{2}{x-4} + \frac{3}{x+1} = 0/(x-4)(x+1)$

$2(x+1)+3(x-4)$
$2x+2+3x-12=0$
$2x+3x=12-2$
$5x=10/:5$
$x= 2$

$\frac{3}{2x+1} + \frac{10}{3x-5} = 0/(2x+1)(3x-5)$

$3(3x-5)+10(2x+1)=0$
$9x-15+20x+10=0$
$29x=5/29$
$x= \frac{5}{29}$

$\frac{2}{x-3} - \frac{5}{x+3} = \frac{1}{x^2-9} /(x-3)(x+3)$

$2(x+3)-5(x-3) = 1$
$2x+5-5x+15= 1$
$-3x=1-5-15$
$-3x=-20/(-3)$
$x= \frac{20}{3}$

$\frac{3}{x} + \frac{2}{x+2} = \frac{5}{x+1}/x(x+2)(x+1)$

$3(x+2)(x+1)+2x(x+1)=5x(x+2)$
$3x+6(x+1)+2x^2+2x=5x^2+10$
$\cancel{3x^2}+6x+7+\cancel{2x^2}+2x=\cancel{5x^2}+10x$
$x=6$

$\frac{2x+1}{x^2-2x} - \frac{x-2}{x^2+2x} = \frac{x+1}{x^2-4}/x(x-2)(x+2)$

$(2x+1)(x+2) - (x-2)(x-2)=x^2+x$
$2x^2+4x+7x+2-(x^2-2x-2x+4)=x^2+x$
$8x= 2$
$x= \frac{1}{4}$

kvadratne jednadžbe

$x^2+2x-8 = 0$

$x_{1,2} = \frac{-2 +- \sqrt{4+32}}{2} = \frac{-2 +- 6}{2}$

$x_1 = 2; x_2 = -4$

bikvadratne jednadžbe

$x^4+3x^2-4=0$
$t=x^2$
$t^2+3t-4=0$
$t_{1,2} = \frac{+3+-\sqrt{9+16}}{2} = \frac{-3+-5}{2}$

$t_1 = 1; t_2= -4; 1= x^2; x_{1,2}= +-\sqrt{1}$
$-4=x^2; x_{3,4} = +-\sqrt{-4} = +-2i$

$x^4-10x^2+9=0$
$t=x^2$
$t^2-10t+9=0$
$t_{1,2}= \frac{10+-\sqrt{100-36}}{2} = \frac{10+-8}{2}$

$t_1=9; t_2=1; x_{1,2}=+-\sqrt{9};x_{1,2}=+-3$
$x_{3,4}= +-\sqrt{1}; x_{3,4}=+-1$

$x^2+2x-8>0$

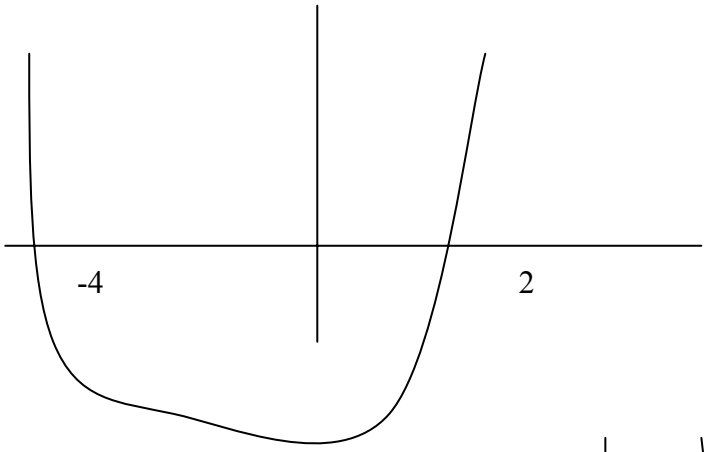
$x^2+2x-8=0$

$$x_{1,2} = \frac{-2+-\sqrt{4+32}}{2} = \frac{2+-6}{2}$$

$x_1= -4;\ x_2=2$

x E <-4,2> ; $N_1(-4,0)$, $N_2(2,0)$

x E <-∞, -4>U<-4, +∞>



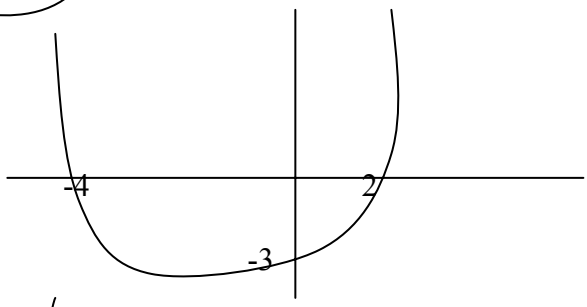
$x^2+2x-8\leq 0$

$x^2+2x-8=0$

$$x_{1,2}= \frac{2+-6}{2}$$

$x_1=2;\ x_2= -4;\ N_1(2,0);\ N_2(-4,0)$

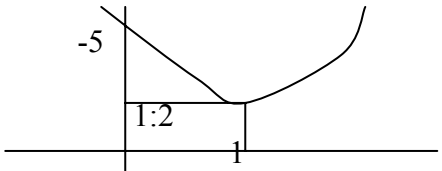
RJ.: x E [-4,2]



$4x^2-8x+5>0$

$$x_{1,2} = \frac{8+-\sqrt{64-80}}{2} = \frac{8+-4i}{8} = \frac{8}{8} +-\frac{4i}{8}$$

$x_1= 1+ \frac{1}{2} i$; x_2 $1- \frac{1}{2}i$



-1:2

$\frac{x+3}{x-7} < -1$

$\frac{x+3}{x-7} + 1<0$

$\frac{x+3+x-7}{x-7} <0$

$\frac{2x-4}{x-7} <0$

	-∞	2	7	+∞
2x-4	-	○		
x-7	-		- ○	+
rj.	+		-	+

2x-4=0 x-7=0

2x=4/2 x=7

x=2

x E <-∞,2>U<7,+∞>

$\frac{x-1}{(2-x)(x-3)} \leq 0$

Rj. x E [1,2> U <3,+∞>

		1		2		3	
x-1	-	●	+		+		+
2-x	+		+	○	-		-
x-3	-		-		-	○	+
Rj.	+		-		-		+