Zbirka nalog za srednje šole: MATEMATIKA

D. Grašek, M. Kožar, A. Tiegl: ELEMENTARNE FUNKCIJE, KOMPLEKSNA ŠTEVILA
Poglavje VII.:Eksponentna funkcija

Str. 55, Reši enačbe

34. a) $0{,}008 = \sqrt[x]{5}$ b) $\sqrt[x-2]{3^x} = \sqrt[x-3]{3^{1+x}}$

c) $\sqrt{5^{3-x}} . \sqrt[3]{25^{x-1}} = 0{,}2$ č) $1000 . 10^x = \sqrt[x]{100^2}$

35. a) $a^{2x} . \sqrt{a^{x-13}} = \sqrt[3]{a^x}$ b) $9^x : \sqrt[3]{3^{x-1}} = 3^{x-1} . \sqrt{3^{3x-4}}$

c) $\sqrt[x-1]{4^{x+1}} : \sqrt[x+1]{4^{x-1}} = \sqrt[3]{4^8}$

36. a) $\sqrt[2x+3]{a^{3x-2}} - \sqrt[2x+4]{a^{3x-5}} = 0$ b) $\sqrt[3x+4]{a^{x-3}} = \sqrt[7x+8]{a^{5x-6}}$

37. a) $4^{x+1} + 4^x = 320$ b) $2 . 3^{x+1} - 4 . 3^{x-2} = 450$

40. a) $7^{x+2} + 2 . 7^{x-1} = 345$ b) $3 . 2^{2x-1} - 9 . 2^x + 5 . 2^{x-2} + 400 = 0$

c) $2 . 3^{x-3} + 3^{x-2} + 3^{x-1} = 14 . 3^5$

Opomba: Priporočam, da najprej rešiš naloge 16, 17, 22, 23 na strani 53 in 54.

Enačbe eksponentne
Pomni:

Rešiti moramo <u>eksponentne enačbe</u> to so enačbe z neznanko v eksponentu.
V osnovi poznamo tri tipe eksponentnih enačb:

(1) $a^{f(x)} = a^{g(x)} \Leftrightarrow f(x) = g(x)$ (pri enakih osnovah izenačimo eksponent)

(2) $a^{f(x)} = b^{f(x)} \Leftrightarrow f(x) = 0$ (Pri enakih eksponentih izenačim eksponent z 0)

(3) $a^{f(x)} = b \Leftrightarrow \log a^{f(x)} = \log b$ (osnovi sta različni in eksponenta sta različna, tedaj enačbo logaritmiramo pri najbolj ugodni osnovi)

Rešitve:

34. a) $0{,}008 = \sqrt[x]{5}$

$$\frac{8}{1000} = 5^{\frac{1}{x}}$$

$$\frac{1}{125} = 5^{\frac{1}{x}}$$

$$5^{-3} = 5^{\frac{1}{x}}$$ (enacim eksponent)

$$\frac{1}{x} = -3 / .x$$ (množim z x)

$$1 = -3x$$

$$\underline{x = -\frac{1}{3}}$$

Uporabim zvezo:

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$

b) $\sqrt[x-2]{3^x} = \sqrt[x-3]{3^{1+x}}$

$$3^{\frac{x}{x-2}} = 3^{\frac{1+x}{x-3}}$$

$$\frac{x}{x-2} = \frac{1+x}{x-3}$$

$$x(x-3) = (x-2)(1+x)$$

$$x^2 - 3x = x + x^2 - 2 - 2x$$

$$-3x = -x - 2$$

$$-2x = -2$$

$$\underline{x = 1}$$

c) $\sqrt{5^{3-x}}.\sqrt[3]{25^{x-1}}=0,2$

$$5^{\frac{3-x}{2}}.5^{\frac{2(x-1)}{3}}=\frac{2}{10}$$

$$5^{\frac{3-x}{2}+\frac{2-x}{3}}=\frac{1}{5}$$

$$5^{\frac{3(3-x)+2(2x-2)}{6}}=5^{-1}$$

$$\frac{3(3-x)+2(2x-2)}{6}=-1/.6$$

$$9-3x+4x-4=-6$$

$$x+5=-6$$

$$\underline{x=-11}$$

č) $1000\,.\,10^{x}=\sqrt[x]{100^{2}}$

$$10^{3}.10^{x}=10^{\frac{4}{x}}$$

$$10^{x+3}=10^{\frac{4}{x}}$$

$$x+3=\frac{4}{x}/.x$$

$$x^{2}+3x=4$$

$$x^{2}+3x-4=0$$

$$(x-1)(x+4)=0$$

$$\underline{x_1=1}\qquad \underline{x_2=-4}$$

35. a) $a^{2x}.\sqrt{a^{x-13}}=\sqrt[3]{a^{x}}$

$$a^{2x}.a^{\frac{x-13}{2}}=a^{\frac{x}{3}}$$

$$a^{2x+\frac{x-13}{2}}=a^{\frac{x}{3}}$$

$$2x+\frac{x-13}{2}=\frac{x}{3}/.6$$

$$12x+3(x-13)=2x$$

$$12x+3x-39=2x$$

$$15x-2x=39$$

$$13x=39$$

$$\underline{x=3}$$

b) $9^{x}:\sqrt[3]{3^{x-1}}=3^{x-1}.\sqrt{3^{3x-4}}$

$$3^{2x}:3^{\frac{x-1}{3}}=3^{x-1}.3^{\frac{3x-4}{2}}$$

$$3^{2x-\frac{x-1}{3}}=3^{x-1+\frac{3x-4}{2}}$$

$$2x-\frac{x-1}{3}=x-1+\frac{3x-4}{2}/.6$$

$$12x-2(x-1)=6x-6+3(3x-4)$$

$$12x-2x+2=6x-6+9x-12$$

$$10x+2=15x-18$$

$$-5x=-20$$

$$\underline{x=4}$$

c) $\sqrt[x-1]{4^{x+1}}:\sqrt[x+1]{4^{x-1}}=\sqrt[3]{4^{8}}$

$$4^{\frac{x+1}{x-1}}:4^{\frac{x-1}{x+1}}=4^{\frac{8}{3}}$$

$$4^{\frac{x+1}{x-1}-\frac{x-1}{x+1}}=4^{\frac{8}{3}}$$

$$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{8}{3}/.3(x-1(x+1)$$

$$3(x+1)^{2}-3(x-1)^{2}=8(x+1)(x-1)$$

$$3(x^{2}+2x+1)-3(x^{2}-2x+1)=8(x^{2}-1)$$

$$3x^{2}+6x+3-3x^{2}+6x-3=8x^{2}-8$$

$$12x=8x^{2}-8$$

$$8x^{2}-12x-8=0/:4$$

$$2x^{2}-3x-2=0$$

$$D=b^{2}-4ac=9+16=25$$

$$x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}=\frac{3\pm5}{4}$$

$$\underline{x_1=2},\qquad \underline{x_2=-\frac{1}{2}}$$

36. a) $\sqrt[2x+3]{a^{3x-2}} - \sqrt[2x+4]{a^{3x-5}} = 0$

$$a^{\frac{3x-2}{2x+3}} - a^{\frac{3x-5}{2x+4}} = 0$$

$$a^{\frac{3x-2}{2x+3}} = a^{\frac{3x-5}{2x+4}}$$

$$\frac{3x-2}{2x+3} = \frac{3x-5}{2x+4}$$

$$(3x-2)(2x+4) = (2x+3)(3x-5)$$

$$6x^2 + 12x - 4x - 8 = 6x^2 - 10x + 9x - 15$$

$$8x - 8 = -x - 15$$

$$9x = -7$$

$$\underline{x = -\frac{7}{9}}$$

b) $\sqrt[3x+4]{a^{x-3}} = \sqrt[7x+8]{a^{5x-6}}$

$$a^{\frac{x-3}{3x+4}} = a^{\frac{5x-6}{7x+8}}$$

$$\frac{x-3}{3x+4} = \frac{5x-6}{7x+8}$$

$$(x-3)(7x+8) = (3x+4)(5x-6)$$

$$7x^2 + 8x - 21x - 24 = 15x^2 - 18x + 20x - 24$$

$$7x^2 - 13x - 24 = 15x^2 + 2x - 24$$

$$8x^2 + 15x = 0$$

$$x(8x+15) = 0$$

$$\underline{x_1 = 0} \qquad 8x + 15 = 0$$

$$\underline{x_2 = -\frac{15}{8}}$$

37. a) $4^{x+1} + 4^x = 320$

$$4^x(4^1 + 4^0) = 320$$

$$4^x(4+1) = 320$$

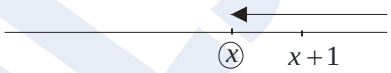
$$4^x.5 = 320 / :5$$

$$4^x = 64$$

$$4^x = 4^3$$

$$\underline{x = 3}$$

Tu si x in $x+1$ zamislimo na številski osi



in izpostavim skupno osnovo na najmanjši eksponent (najbolj levo na osi). Tu je to x.

b) $2 . 3^{x+1} - 4 . 3^{x-2} = 450$

$$3^{x-2}(2 . 3^3 - 4 . 3^0) = 450$$

$$3^{x-2}(54 - 4) = 450$$

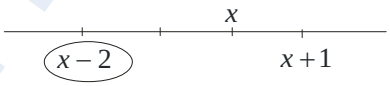
$$3^{x-2}. 50 = 450$$

$$3^{x-2} = 9$$

$$3^{x-2} = 3^2$$

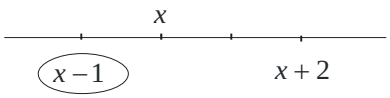
$$x - 2 = 2$$

$$\underline{x = 4}$$



40. a) $7^{x+2}+2\,.\,7^{x-1}=345$

$$7^{x-1}(7^3+2\,.\,7^0)=235$$
$$7^{x-1}(343+2)=345$$
$$7^{x-1}=1$$
$$7^{x-1}=7^0$$
$$x-1=0$$
$$\underline{x=1}$$



b) $3\,.\,2^{2x-1}-9\,.\,2^x+5\,.\,2^{x-2}+400=0$

$$3.2^{x-1}-9\,.\,2^x+5\,.\,2^{x-2}=-400$$
$$2^{x-2}(3\,.\,2-9\,.\,2^2+5\,.\,2^0)=-400$$
$$2^{x-2}(6-36+5)=-400$$
$$-25\,.\,2^{x-2}=-400$$
$$2^{x-2}=16$$
$$2^{x-2}=2^4$$
$$x-2=4$$
$$\underline{x=6}$$

Člen brez faktorja 2^{x+n} dam na desno! Potem lahko na levi izpostavim 2 na najmanjši skupni eksponent.

c) $2\,.\,3^{x-3}+3^{x-2}+3^{x-1}=14\,.\,3^5$

$$3^{x-3}(2+3+9)=14\,.\,3^5$$
$$3^{x-3}(2+3+9)=14\,.\,3^5$$
$$3^{x-3}\,.\,14=14\,.\,3^5$$
$$3^{x-3}=3^5$$
$$x-3=5$$
$$\underline{x=8}$$