



$$\left\{ \begin{array}{l} (3e)^x = u \\ 3^x e^x (1 + \ln 3) dx = du \end{array} \right.$$

$$= \int (3e)^x \frac{du}{3^x e^x (1 + \ln 3)} = \int \frac{du}{(1 + \ln 3)} = \frac{1}{1 + \ln 3} u = \frac{(3e)^x}{1 + \ln 3} + C$$

1051\*\*

$$\int \frac{adx}{a-x} = a \int \frac{dx}{a-x}$$

$$\left\{ \begin{array}{l} a-x = u \\ -dx = du \end{array} \right.$$

$$= a \int \frac{-du}{u} = -a \ln u = -a \ln(a-x) + C$$

1052\*\*

$$\int \frac{2x+3}{2x+1} dx = \int \frac{2x+1+2}{2x+1} dx = \int dx + \int \frac{2}{2x+1} dx = x + \int \frac{2}{2x+1} dx$$

$$\left\{ \begin{array}{l} 2x+1 = u \\ 2dx = du \end{array} \right.$$

$$x + \int \frac{du}{u} = x + \ln u = x + \ln(2x+1) + C$$

1054

$$\int \frac{xdx}{a+bx} = \frac{1}{b} \int \frac{bxdx}{a+bx} = \frac{1}{b} \int \frac{bx+a-a}{a+bx} dx = \frac{1}{b} \int \frac{bx+a-a}{a+bx} dx = \frac{1}{b} \int dx - \frac{a}{b} \int \frac{dx}{a+bx}$$

$$= \frac{x}{b} - \frac{a}{b} \int \frac{dx}{a+bx}$$

$$\left\{ \begin{array}{l} a+bx = u \\ bdx = du \end{array} \right.$$

$$= \frac{x}{b} - \frac{a}{b} \int \frac{du}{u} = \frac{x}{b} - \frac{a}{b^2} \int \frac{du}{u} = \frac{x}{b} - \frac{a}{b^2} \ln u = \frac{x}{b} - \frac{a}{b^2} \ln(a+bx) + C$$

1055

$$\int \frac{ax+b}{ax+\beta} dx = a \int \frac{x+\frac{b}{a}}{ax+\beta} dx = a \int \frac{xdx}{ax+\beta} + \int \frac{\frac{b}{a}}{ax+\beta} dx$$

$$= \frac{a}{\alpha} \int \frac{ax}{ax+\beta} dx + b \int \frac{dx}{ax+\beta} = \frac{a}{\alpha} \int \frac{ax+\beta-\beta}{ax+\beta} dx + b \int \frac{dx}{ax+\beta}$$

$$= \frac{a}{\alpha} \int dx + \frac{\beta}{\alpha} \int \frac{-\beta}{ax+\beta} dx + b \int \frac{dx}{ax+\beta} = \frac{ax}{\alpha} - \frac{a\beta}{\alpha} \int \frac{dx}{ax+\beta} + b \int \frac{dx}{ax+\beta}$$

$$= \frac{ax}{\alpha} + \left( b - \frac{a\beta}{\alpha} \right) \int \frac{dx}{ax+\beta} = \frac{ax}{\alpha} + \frac{ba-\beta a}{\alpha} \int \frac{dx}{ax+\beta}$$

$$\left\{ \begin{array}{l} ax+\beta = u \\ adx = du \end{array} \right.$$

$$= \frac{ax}{\alpha} + \frac{ba-\beta a}{\alpha} \int \frac{du}{u} = \frac{ax}{\alpha} + \frac{ba-\beta a}{\alpha^2} \int \frac{du}{u} = \frac{ax}{\alpha} + \frac{ba-\beta a}{\alpha^2} \ln u = a \frac{x}{\alpha} + \frac{ba-\beta a}{\alpha^2} \ln(ax+\beta) + C$$

1056

$$\int \frac{x^2+1}{x-1} dx : (x-1) = x+1 + \frac{2}{(x-1)}$$

$$x^2 - x$$

1 + x

x - 1

$$2$$

$$= \int x dx + \int dx + 2 \int \frac{dx}{(x-1)} = \frac{x^2}{2} + x + 2 \ln(x-1) + C$$

1057

$$\int \frac{x^2+5x+7}{x+3} dx = \int \frac{x^2}{x+3} dx + 5 \int \frac{x}{x+3} dx + 7 \int \frac{dx}{x+3} = \int \frac{x^2+9-9}{x+3} dx + 5 \int \frac{x+3-3}{x+3} dx + 7 \ln(x+3)$$

$$= \int (x-3) dx + 9 \int \frac{dx}{x+3} + 5 \int dx - 15 \int \frac{dx}{x+3} + 7 \ln(x+3)$$

$$= \frac{x^2}{2} - 3x + 9 \ln(x+3) + 5x - 15 \ln(x+3) + 7 \ln(x+3)$$

$$= \frac{1}{2}x^2 + 2x + \ln(x+3) + C$$

1058

$$\int \frac{x^4+x^2+1}{x-1} dx = \int \frac{x^4}{x-1} dx + \int \frac{x^2}{x-1} dx + \int \frac{dx}{x-1} = \int \frac{x^4+1-1}{x-1} dx + \int \frac{x^2+1-1}{x-1} dx + \ln(x-1)$$

$$= \int \frac{x^4-1}{x-1} dx + \ln(x-1) + \int \frac{x^2-1}{x-1} dx + \ln(x-1) + \ln(x-1)$$

$$= 3 \ln(x-1) + \int \frac{(x^2+1)(x+1)(x-1)}{x-1} dx + \int \frac{(x+1)(x-1)}{x-1} dx$$

$$= 3 \ln(x-1) + \int (x+1) dx + \int (x^2+1)(x+1) dx$$

$$= 3 \ln(x-1) + \frac{x^2}{2} + x + \int (x^3+x^2+x+1) dx = 3 \ln(x-1) + \frac{x^2}{2} + x + \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x$$

$$= \frac{1}{4}x^4 + \frac{1}{3}x^3 + x^2 + 2x + 3 \ln(x-1) + C$$

1059

$$\int (a + \frac{b}{x-a})^2 dx = \int a^2 dx + 2 \int \frac{ba}{x-a} dx + \int \frac{b^2}{(x-a)^2} dx = a^2 x + 2ab \ln(x-a) + b^2 \int \frac{dx}{(x-a)^2}$$

$$\left\{ \begin{array}{l} x-a = t \\ dx = dt \end{array} \right.$$

$$= a^2 x + 2ab \ln(x-a) + b^2 \int \frac{dt}{t^2}$$

$$= xa^2 + 2ab \ln(x-a) - \frac{b^2}{t} = xa^2 + 2ab \ln(x-a) - \frac{b^2}{x-a} + C$$

1060\*

$$\int \frac{dx}{(x+1)^2} dx = \int \frac{x+1-1}{(x+1)^2} dx = \int \frac{dx}{x+1} - \int \frac{dx}{(x+1)^2} = \ln(x+1) - \int \frac{dx}{(x+1)^2}$$

$$\left\{ \begin{array}{l} x+1 = t \\ dx = dt \end{array} \right.$$

$$= \ln(x+1) - \int \frac{dt}{t^2} = \ln(x+1) + \frac{1}{t} = \ln(x+1) + \frac{1}{x+1} + C$$

1061

$$\int \frac{bdy}{\sqrt{1-y}} = b \int \frac{dy}{\sqrt{1-y}}$$

$$\left\{ \begin{array}{l} 1-y = t \\ -dy = dt \end{array} \right.$$

$$= -b \int \frac{dt}{\sqrt{t}} = -2b\sqrt{t} = -2b\sqrt{1-y} + C$$

1062

$$\int \sqrt{a-bx} dx$$



$$\begin{aligned} & \int \frac{x dx}{x^2 - 5} \\ & \left\{ \begin{array}{l} x^2 - 5 = t' \\ 2x dx = dt \end{array} \right\} \\ & = \int \frac{\frac{dt}{t}}{t} = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t = \frac{1}{2} \ln(x^2 - 5) + C \end{aligned}$$

1078

$$\begin{aligned} & \int \frac{x dx}{2x^2 + 3} \\ & \left\{ \begin{array}{l} 2x^2 + 3 = t \\ 4x dx = dt \end{array} \right\} \\ & = \int \frac{\frac{dt}{4}}{t} = \frac{1}{4} \int \frac{dt}{t} = \frac{1}{4} \ln t = \frac{1}{4} \ln(2x^2 + 3) + C \end{aligned}$$

1079

$$\begin{aligned} & \int \frac{ax+b}{a^2x^2+b^2} dx \quad (a > 0) = \frac{1}{a} \int \frac{ax+b}{ax^2+\frac{b^2}{a}} dx = \frac{1}{a} \int \frac{ax}{ax^2+\frac{b^2}{a}} dx + \frac{b}{a^2} \int \frac{dx}{x^2+\frac{b^2}{a^2}} \\ & = \frac{1}{a} \int \frac{ax}{ax^2+\frac{b^2}{a}} dx + \frac{b}{a^2} \int \frac{dx}{x^2+\frac{b^2}{a^2}} \\ & \left\{ \begin{array}{l} ax^2 + \frac{b^2}{a} = t' \\ 2ax dx = dt \end{array} \right\} \\ & = \frac{1}{a} \int \frac{\frac{dt}{t}}{t} dx + \frac{b}{a^2} * \frac{a}{b} \arctan \frac{a}{b} x = \frac{1}{2a} \int \frac{dt}{t} + \frac{1}{a} \arctan \frac{a}{b} x \\ & = \frac{1}{2a} \ln(ax^2 + \frac{b^2}{a}) + \frac{1}{a} \arctan \frac{a}{b} x + C \end{aligned}$$

1080

$$\begin{aligned} & \int \frac{x dx}{\sqrt{a^4-x^4}} \\ & \left\{ \begin{array}{l} x^4 = t^2 |^{\frac{1}{2}} \\ x^2 = t' \\ 2x dx = dt \end{array} \right\} \\ & = \int \frac{\frac{dt}{2}}{\sqrt{a^4-t^2}} = \frac{1}{2} \int \frac{dt}{\sqrt{a^4-t^2}} = \frac{1}{2} \arctan \frac{t}{\sqrt{(a^4-t^2)}} = \frac{1}{2} \arctan \frac{x^2}{\sqrt{(a^4-x^4)}} + C \end{aligned}$$

1081

$$\begin{aligned} & \int \frac{x^2 dx}{1+x^6} \\ & \left\{ \begin{array}{l} x^6 = t^2 |^{\frac{1}{2}} \\ x^3 = t' \\ 3x^2 dx = dt \end{array} \right\} \\ & = \int \frac{\frac{dt}{3}}{1+t^2} = \frac{1}{3} \int \frac{dt}{1+t^2} = \frac{1}{3} \arctan t = \frac{1}{3} \arctan x^3 + C \end{aligned}$$

1082

$$\int \frac{x^2 dx}{\sqrt{x^6-1}}$$

$$\begin{aligned} & \left\{ \begin{array}{l} x^6 = t^2 |^{\frac{1}{2}} \\ x^3 = t' \\ 3x^2 dx = dt \end{array} \right\} \\ & = \int \frac{\frac{dt}{3}}{\sqrt{t^2-1}} = \frac{1}{3} \int \frac{dt}{\sqrt{t^2-1}} = \frac{1}{3} \ln(t + \sqrt{t^2-1}) = \frac{1}{3} \ln(x^3 + \sqrt{(x^6-1)}) + C \end{aligned}$$

1083

$$\int \sqrt{\frac{\arcsin x}{1-x^2}} dx = \left\{ \begin{array}{l} \arcsin x = u' \\ \frac{dx}{\sqrt{1-x^2}} = du \end{array} \right\} = \int \sqrt{u} du = \frac{2}{3} u^{\frac{3}{2}} = \frac{2}{3} \arcsin x^{\frac{3}{2}} + C$$

1084

$$\begin{aligned} & \int \frac{\arctan \frac{x}{2}}{x^2+4} dx \\ & \left\{ \begin{array}{l} \arctan \frac{x}{2} = u' \\ \frac{2dx}{x^2+4} = du \end{array} \right\} \\ & = \frac{1}{2} \int u du = \frac{u^2}{4} = \frac{1}{4} \arctan^2 \frac{x}{2} + C \end{aligned}$$

1085

$$\begin{aligned} & \int \frac{x \sqrt{\arctan 2x}}{1+4x^2} dx = \int \frac{x}{1+4x^2} dx - \int \frac{\sqrt{\arctan 2x}}{1+4x^2} dx \\ & \left\{ \begin{array}{l} 1+4x^2 = u' \\ 8x dx = du \end{array} \right\} \left\{ \begin{array}{l} = t \\ \frac{2dx}{1+4x^2} = dt \end{array} \right\} \\ & = \int \frac{\frac{du}{8}}{u} - \int \sqrt{t} \frac{dt}{2} = \frac{1}{8} \ln u - \frac{1}{2} * \frac{2}{3} t^{\frac{3}{2}} = \frac{1}{8} \ln u - \frac{1}{3} t^{\frac{3}{2}} = \end{aligned}$$

$$\frac{1}{8} \ln(1+4x^2) - \frac{1}{3} \arctan^{\frac{3}{2}} 2x + C$$

1086

$$\begin{aligned} & \int \frac{dx}{\sqrt{(1+x^2) \ln(x+\sqrt{1+x^2})}} \\ & \left\{ \begin{array}{l} \ln(x+\sqrt{1+x^2}) = u' \\ \frac{dx}{\sqrt{x^2+1}} = du \end{array} \right\} \\ & = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} = 2\sqrt{\ln(x+\sqrt{1+x^2})} + C \end{aligned}$$

1087

$$\begin{aligned} & \int ae^{-mx} dx \\ & \left\{ \begin{array}{l} -mx = u' \\ -mdx = du \end{array} \right\} \\ & = \int ae^u (-\frac{du}{m}) = -\frac{a}{m} \int e^u du = -\frac{a}{m} e^u = -\frac{a}{m} e^{-mx} + C \end{aligned}$$

1088

$$\int 4^{2-3x} dx =$$

$$\begin{aligned} & \left\{ \begin{array}{l} 2 - 3x = t' \\ -3dx = dt \end{array} \right\} \left\{ \begin{array}{l} 4^t = u' \\ 4^t \ln 4dt = du \end{array} \right\} \\ & = \int 4^t \frac{dt}{-3} = -\frac{1}{3} \int 4^t dt \\ & = -\frac{1}{3} \int 4^t \frac{du}{4^t \ln 4} = -\frac{1}{3 \ln 4} \int du = -\frac{1}{3 \ln 4} u = -\frac{4^t}{3 \ln 4} = -\frac{1}{3 \ln 4} 4^{2-3x} + C \end{aligned}$$

1089

$$\begin{aligned} \int (e^t - e^{-t})dt &= 2 \int \frac{e^t - e^{-t}}{2} dt = 2 \int \sinh t dt = 2 \cosh t + C \\ \int (e^t - e^{-t})dt &= \int e^t dt - \int e^{-t} dt = e^t + e^{-t} + C \end{aligned}$$

1090

$$\begin{aligned} \int (e^{\frac{x}{a}} + e^{-\frac{x}{a}})^2 dx &= \int e^{2\frac{x}{a}} dx + 2 \int dx + \int \frac{dx}{e^{2\frac{x}{a}}} \\ &\left\{ \begin{array}{l} 2\frac{x}{a} = t' \\ \frac{2dx}{a} = dt \end{array} \right\} \\ &= \int e^t \frac{a}{2} dt + 2x + \int \frac{a}{2} e^{-t} dt = \frac{a}{2} \int e^t dt + 2x + \frac{a}{2} \int e^{-t} dt \\ &= \frac{1}{2} ae^{2\frac{x}{a}} + 2x - \frac{1}{2} ae^{-2\frac{x}{a}} + C \end{aligned}$$

1091

$$\begin{aligned} \int \frac{(a^x - b^x)^2}{a^x b^x} dx &= \int \frac{a^{2x} - 2a^x b^x + b^{2x}}{a^x b^x} dx = \int \frac{a^x}{b^x} dx - \int 2dx + \int \frac{b^x}{a^x} dx \\ &= \int \left(\frac{a}{b}\right)^x dx - 2x + \int \left(\frac{b}{a}\right)^x dx = \frac{\left(\frac{a}{b}\right)^x}{\ln \frac{a}{b}} - 2x + \frac{\left(\frac{b}{a}\right)^x}{\ln \frac{b}{a}} + C \end{aligned}$$

1092

$$\begin{aligned} \int \frac{a^{2x-1}}{\sqrt{a^x}} dx &= \int a^{2x-\frac{1}{2}x} dx - \int a^{-\frac{1}{2}x} dx = \int a^{\frac{3}{2}x} dx - \int a^{-\frac{1}{2}x} dx = \int a^u \frac{2}{3} du - \int a^t (-2) dt \\ &\left\{ \begin{array}{l} \frac{3}{2}x = u' \quad -\frac{1}{2}x = t' \\ \frac{3}{2}dx = du \quad -\frac{dx}{2} = dt \end{array} \right\} \\ &= \frac{2}{3 \ln a} a^u + \frac{2}{\ln a} a^t = \frac{2}{\ln a} \left( \frac{a^{\frac{3}{2}x}}{3} + a^{-\frac{1}{2}x} \right) + C \end{aligned}$$

1093

$$\begin{aligned} \int (e^{-(x^2+1)}) x dx &= e^{-x^2-1} x dx \\ &\left\{ \begin{array}{l} -x^2 - 1 = t' \\ -2x dx = dt \end{array} \right\} \\ &= \int e^t \frac{dt}{-2} = -\frac{1}{2} \int e^t dt = -\frac{1}{2} e^t \\ &= -\frac{1}{2} e^{-x^2-1} + C \end{aligned}$$

1094

$$\begin{aligned} \int x 7^{x^2} dx &\\ &\left\{ \begin{array}{l} x^2 = t' \\ 2x dx = dt \end{array} \right\} \\ &= \int 7^t \frac{dt}{2} = \frac{1}{2} \int 7^t dt = \frac{1}{2 \ln 7} 7^t = \frac{1}{2 \ln 7} 7^{x^2} + C \end{aligned}$$

1095

$$\begin{aligned} \int \frac{e^{\frac{1}{x}}}{x^2} dx &\\ &\left\{ \begin{array}{l} \frac{1}{x} = t' \\ -\frac{dx}{x^2} = dt \end{array} \right\} \\ &= - \int e^t dt = -e^t = -e^{\frac{1}{x}} + C \end{aligned}$$

1096

$$\begin{aligned} \int 5\sqrt{x} \frac{dx}{\sqrt{x}} &\\ &\left\{ \begin{array}{l} \sqrt{x} = t' \\ \frac{dx}{2\sqrt{x}} = dt \end{array} \right\} \\ &= \int 5^t 2dt = 2 \int 5^t dt = \frac{2}{\ln 5} 5^t = \frac{2}{\ln 5} 5\sqrt{x} + C \end{aligned}$$

1097

$$\begin{aligned} \int \frac{e^x}{e^x - 1} dx &\\ &\left\{ \begin{array}{l} e^x - 1 = t' \\ e^x dx = dt \end{array} \right\} \\ &= \int \frac{dt}{t} = \ln t = \ln(e^x - 1) + C \end{aligned}$$

1098

$$\begin{aligned} \int e^x \sqrt{a - be^x} dx &\\ &\left\{ \begin{array}{l} a - be^x = t' \\ -be^x dx = dt \end{array} \right\} \\ &= \int \sqrt{t} \frac{dt}{-b} = -\frac{1}{b} \int \sqrt{t} dt = -\frac{2}{3b} t^{\frac{3}{2}} = -\frac{2}{3b} (a - be^x)^{\frac{3}{2}} + C \end{aligned}$$

1099

$$\begin{aligned} \int (e^{\frac{x}{a}} + 1)^{\frac{1}{3}} e^{\frac{x}{a}} dx &\\ &\left\{ \begin{array}{l} e^{\frac{x}{a}} + 1 = t' \\ \frac{1}{a} e^{\frac{x}{a}} dx = dt \end{array} \right\} \\ &= \int t^{\frac{1}{3}} a dt = a \int t^{\frac{1}{3}} dt = \frac{3}{4} a t^{\frac{4}{3}} = \frac{3}{4} a (e^{\frac{x}{a}} + 1)^{\frac{4}{3}} + C \end{aligned}$$

1100\*

$$\begin{aligned} \int \frac{dx}{2^{2+3}} &\\ &\left\{ \begin{array}{l} 2^x + 3 = t' \\ 2^x \ln 2 dx = dt \end{array} \right\} \\ &= \int \frac{\frac{dt}{2^x \ln 2}}{t} = \frac{1}{\ln 2} \int \frac{dt}{2^x t} = \frac{1}{\ln 2} \int \frac{dt}{(t-3)t} = \frac{1}{\ln 2} \int \frac{dt}{t^2-3t} = \frac{1}{\ln 2} \int \frac{dt}{(t-\frac{3}{2})^2-\frac{9}{4}} \\ &= \frac{1}{\ln 2} * \frac{1}{\sqrt{\frac{9}{4}}} \operatorname{arctanh} \frac{t-\frac{3}{2}}{\sqrt{\frac{9}{4}}} = \frac{2}{3 \ln 2} \operatorname{arctanh} \left( \frac{2}{3} t - 1 \right) = \end{aligned}$$

$$\frac{2}{3 \ln 2} \operatorname{arctanh}\left(\frac{2}{3}(2^x + 3) - 1\right) + C$$

1101

$$\begin{aligned} & \int \frac{a^x dx}{1+a^{2x}} \\ & \left\{ \begin{array}{l} a^x = u \\ a^x \ln a dx = du \end{array} \right\} \\ & = \int \frac{du}{\ln a(1+u^2)} = \frac{1}{\ln a} \int \frac{du}{1+u^2} = \frac{1}{\ln a} \arctan u = \frac{1}{\ln a} \arctan a^x + C \end{aligned}$$

1102

$$\begin{aligned} & \int \frac{e^{-bx}}{1-e^{-2bx}} dx \\ & \left\{ \begin{array}{l} e^{-bx} = u \\ -be^{-bx} dx = du \end{array} \right\} \\ & = \int \frac{du}{1-u^2} = -\frac{1}{b} \int \frac{du}{1-u^2} = -\frac{1}{b} \operatorname{arctanh} u = -\frac{1}{b} \operatorname{arctanh} e^{-bx} + C \end{aligned}$$

1103

$$\begin{aligned} & \int \frac{e^t}{\sqrt{1-e^{2t}}} dt \\ & \left\{ \begin{array}{l} e^t = u \\ e^t dt = du \end{array} \right\} \\ & = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u = \arcsin e^t + C \end{aligned}$$

1104

$$\begin{aligned} & \int \sin(a+bx) dx \\ & \left\{ \begin{array}{l} a+bx = z \\ bdx = dz \end{array} \right\} \\ & = \int \sin z \frac{dz}{b} = \frac{1}{b} \int \sin z dz = -\frac{1}{b} \cos z = -\frac{1}{b} \cos(a+bx) + C \end{aligned}$$

1105

$$\begin{aligned} & \int \cos \frac{x}{\sqrt{2}} dx \\ & \left\{ \begin{array}{l} \frac{x}{\sqrt{2}} = z \\ \frac{dx}{\sqrt{2}} = dz \end{array} \right\} \\ & = \int \cos z \sqrt{2} dz = \sqrt{2} \int \cos z dz = \sqrt{2} \sin z = \sqrt{2} \sin\left(\frac{x}{\sqrt{2}}\right) + C \end{aligned}$$

1106

$$\begin{aligned} & \int (\cos ax + \sin ax)^2 dx = \int (\sin^2 ax + 2 \sin ax \cos ax + \cos^2 ax) dx = \int (1 + 2 \sin ax \cos ax) dx \\ & = \int (1 + \sin 2ax) dx = x + \int \sin 2ax dx \\ & \left\{ \begin{array}{l} 2ax = z \\ 2adx = dz \end{array} \right\} \\ & = x + \int \sin z \frac{dz}{2a} = x + \frac{1}{2a} \int \sin z dz = x - \frac{1}{2a} \cos z = x - \frac{1}{2a} \cos 2ax + C \end{aligned}$$

1107

$$\begin{aligned} & \int \cos \sqrt{x} \frac{dx}{\sqrt{x}} \\ & \left\{ \begin{array}{l} \sqrt{x} = z \\ \frac{dx}{2\sqrt{x}} = dz \end{array} \right\} \\ & = \int \cos z 2dz = 2 \int \cos z dz = 2 \sin z = 2 \sin \sqrt{x} + C \end{aligned}$$

1108

$$\begin{aligned} & \int \sin(\lg x) \frac{dx}{x} = \int \sin\left(\frac{\ln x}{\ln 2}\right) \frac{dx}{x} \\ & \left\{ \begin{array}{l} \frac{\ln x}{\ln 2} = z \\ \frac{dx}{x \ln 2} = dz \end{array} \right\} \\ & = \int \sin(z) \ln 2 dz = \ln 2 \int \sin z dz \\ & = -\ln 2 \cos z = -\ln 2 \cos \frac{\ln x}{\ln 2} = -\ln 2 \cos \lg x + C \end{aligned}$$

1109

$$\begin{aligned} & \int \sin^2 x dx = \int \frac{1-\cos 2x}{2} dx = \frac{1}{2} \int dx - \frac{1}{2} \int \cos 2x dx \\ & \left\{ \begin{array}{l} 2x = z \\ 2dx = dz \end{array} \right\} \\ & = \frac{x}{2} - \frac{1}{2} \int \cos z \frac{dz}{2} = \frac{x}{2} - \frac{1}{4} \int \cos z dz = \frac{1}{2}x - \frac{1}{4} \sin z = \frac{1}{2}x - \frac{1}{4} \sin 2x + C \end{aligned}$$

1110\*

$$\begin{aligned} & \int \cos^2 x dx = \int \left(\sqrt{\frac{1}{2}(1+\cos 2x)}\right)^2 dx = \int \frac{1}{2} dx + \int \frac{1}{2} \cos 2x dx = \frac{1}{2}x + \frac{1}{2} \int \cos 2x dx \\ & \left\{ \begin{array}{l} 2x = z \\ 2dx = dz \end{array} \right\} \\ & = \frac{1}{2}x + \frac{1}{2} \int \cos z \frac{dz}{2} = \frac{1}{2}x + \frac{1}{4} \int \cos z dz = \frac{1}{2}x + \frac{1}{4} \sin z = \frac{1}{2}x + \frac{1}{4} \sin 2x + C \end{aligned}$$

1111

$$\begin{aligned} & \int \sec^2(ax+b) dx = \int \frac{dx}{\cos^2(ax+b)} \\ & \left\{ \begin{array}{l} ax+b = t \\ a dx = dt \end{array} \right\} \\ & \int \frac{\frac{dt}{a}}{\cos^2 t} = \frac{1}{a} \int \frac{dt}{\cos^2 t} = \frac{1}{a} \tan t = \frac{1}{a} \tan(ax+b) + C \end{aligned}$$

1112

$$\begin{aligned} & \int \cot^2 ax dx = \int \frac{\cos^2 ax}{\sin^2 ax} dx = \int \frac{1-\sin^2 ax}{\sin^2 ax} dx = \int \frac{dx}{\sin^2 ax} - \int dx \\ & \left\{ \begin{array}{l} ax = t \\ a dx = dt \end{array} \right\} \\ & = \int \frac{\frac{dt}{a}}{\sin^2 t} - x = \frac{1}{a} \int \frac{dt}{\sin^2 t} - x = -\frac{1}{a} \cot t - x = -\frac{1}{a} \cot(ax) - x + C \end{aligned}$$

1113

$$\begin{aligned} & \int \frac{dx}{\sin \frac{x}{a}} \\ & \left\{ \begin{array}{l} \frac{x}{a} = t \\ \frac{dx}{a} = dt \end{array} \right\} \\ & \int \frac{adt}{\sin t} = a \int \frac{dt}{\sin t} = a \ln \tan \frac{t}{2} = a \ln \tan \frac{\frac{x}{a}}{2} = a \ln(\tan \frac{x}{2a}) + C \end{aligned}$$

1114

$$\begin{aligned} & \int \frac{dx}{3 \cos(5x - \frac{\pi}{4})} \\ & \left\{ \begin{array}{l} 5x - \frac{\pi}{4} = t \\ 5dx = dt \end{array} \right\} \\ & = \int \frac{dt}{3 \cos t} = \frac{1}{15} \int \frac{dt}{\cos t} = \frac{1}{15} \ln \tan(\frac{t}{2} + \frac{\pi}{4}) \\ & = \frac{1}{15} \ln(\tan(\frac{1}{2}(5x - \frac{\pi}{4}) + \frac{1}{4}\pi)) = \frac{1}{15} \ln(\tan(\frac{5}{2}x + \frac{1}{8}\pi)) + C \end{aligned}$$

1115

$$\begin{aligned} & \int \frac{dx}{\sin(ax+b)} \\ & \left\{ \begin{array}{l} ax + b = t \\ adx = dt \end{array} \right\} \\ & = \int \frac{dt}{\sin t} = \frac{1}{a} \int \frac{dt}{\sin t} = \frac{1}{a} \ln \tan \frac{t}{2} = \frac{1}{a} \ln \tan \frac{ax+b}{2} + C \end{aligned}$$

1116

$$\begin{aligned} & \int \frac{xdx}{\cos^2 x^2} \\ & \left\{ \begin{array}{l} x^2 = t \\ 2xdx = dt \end{array} \right\} \\ & = \int \frac{dt}{\cos^2 t} = \frac{1}{2} \int \frac{dt}{\cos^2 t} = \frac{1}{2} \tan t = \frac{1}{2} \tan x^2 + C \end{aligned}$$

1117

$$\begin{aligned} & \int x \sin(1-x^2) dx \\ & \left\{ \begin{array}{l} 1-x^2 = t \\ -2xdx = dt \end{array} \right\} \\ & = -\int \sin t \frac{dt}{2} = -\frac{1}{2} \int \sin t dt = \frac{1}{2} \cos t = \frac{1}{2} \cos(1-x^2) + C \end{aligned}$$

1118

$$\begin{aligned} & \int \left( \frac{1}{\sin x \sqrt{2}} - 1 \right)^2 dx = \int \left( \frac{1}{\sin^2 x \sqrt{2}} - \frac{2}{\sin x \sqrt{2}} + 1 \right) dx = x - 2 \int \frac{dx}{\sin x \sqrt{2}} + \int \frac{dx}{\sin^2 x \sqrt{2}} \\ & \left\{ \begin{array}{l} x\sqrt{2} = t \\ \sqrt{2} dx = dt \end{array} \right\} \\ & = x - 2 \int \frac{dt}{\sin t} + \int \frac{dt}{\sin^2 t} = x - \sqrt{2} \int \frac{dt}{\sin t} + \frac{\sqrt{2}}{2} \int \frac{dt}{\sin^2 t} = x - \sqrt{2} \ln \tan \frac{t}{2} - \frac{\sqrt{2}}{2} \cot t \\ & = x - \sqrt{2} \ln \left( \tan \frac{x\sqrt{2}}{2} \right) - \frac{\sqrt{2}}{2} \cot x \sqrt{2} + C \end{aligned}$$

1119

$$\begin{aligned} & \int \tan x dx = \int \frac{\sin x}{\cos x} dx = \int \frac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}} dx = \int \frac{\sqrt{1-\cos 2x}}{\sqrt{1+\cos 2x}} \frac{\sqrt{1+\cos 2x}}{\sqrt{1+\cos 2x}} dx = \\ & = \int \frac{\sqrt{1-\cos^2 2x}}{1+\cos 2x} dx = \int \frac{\sqrt{\sin^2 2x}}{1+\cos 2x} dx = \int \frac{\sin 2x}{1+\cos 2x} dx \\ & \left\{ \begin{array}{l} 1+\cos 2x = t' \\ -2 \sin 2x dx = dt \end{array} \right\} \\ & = \int \frac{-dt}{t'} = -\frac{1}{2} \ln t = -\frac{1}{2} \ln(1+\cos 2x) + C \end{aligned}$$

1120

$$\begin{aligned} & \int \cot x dx = \int \frac{\cos x}{\sin x} dx = \int \sqrt{\frac{\frac{1}{2}(1+\cos 2x)}{\frac{1}{2}(1-\cos 2x)}} dx = \int \frac{\sqrt{1+\cos 2x}}{\sqrt{1-\cos 2x}} dx = \int \frac{\sqrt{1+\cos 2x}}{\sqrt{1-\cos 2x}} \frac{\sqrt{1-\cos 2x}}{\sqrt{1-\cos 2x}} dx = \int \frac{\sqrt{1-\cos^2 2x}}{1-\cos 2x} \\ & \int \frac{\sin 2x}{1-\cos 2x} dx \\ & \left\{ \begin{array}{l} 1-\cos 2x = t' \\ 2 \sin 2x dx = dt \end{array} \right\} \\ & \int \frac{dt}{t'} = \frac{1}{2} \ln t = \frac{1}{2} \ln(1-\cos 2x) + C \end{aligned}$$

1121

$$\begin{aligned} & \int \cot \frac{x}{a-b} dx \\ & \left\{ \begin{array}{l} \frac{x}{a-b} = u' \\ \frac{dx}{a-b} = du \end{array} \right\} \\ & = \int \cot u (a-b) du = (a-b) \int \cot u du = (a-b) \ln(\sin u) = (a-b) \ln \left( \sin \left( \frac{x}{a-b} \right) \right) + C \end{aligned}$$

1122

$$\begin{aligned} & \int \frac{dx}{\tan \frac{x}{5}} = \int \cot \frac{x}{5} dx \\ & \left\{ \begin{array}{l} \frac{x}{5} = u' \\ \frac{dx}{5} = du \end{array} \right\} \\ & = \int \cot u 5 du = 5 \int \frac{\cos u}{\sin u} du \\ & \left\{ \begin{array}{l} \sin u = t' \\ \cos u du = dt \end{array} \right\} \\ & = 5 \int \frac{dt}{t} = 5 \ln t = -5 \ln(\sin u) = 5 \ln \sin \frac{x}{5} + C \end{aligned}$$

1123

$$\begin{aligned} & \int \tan \sqrt{x} \frac{dx}{\sqrt{x}} \\ & \left\{ \begin{array}{l} \sqrt{x} = u' \\ \frac{dx}{2\sqrt{x}} = du \end{array} \right\} \\ & = 2 \int \tan u du = 2 \int \tan u = 2 \int \frac{\sin u}{\cos u} du \end{aligned}$$

$$\begin{cases} \cos u = t|' \\ -\sin u du = dt \end{cases}$$

$$= -2 \int \frac{dt}{t} = -2 \ln t = -2 \ln(\cos u) = -2 \ln \cos \sqrt{x} + C$$

1124

$$\int x \cot(x^2 + 1) dx$$

$$\begin{cases} x^2 + 1 = u \\ 2x dx = du \end{cases}$$

$$= \int \cot u \frac{du}{2} = \frac{1}{2} \int \cot u du = \frac{1}{2} \ln(\sin u) = \frac{1}{2} \ln \sin(x^2 + 1) + C$$

1125

$$\int \frac{dx}{\sin x \cos x} = \int \frac{dx}{\frac{\sin 2x}{2}} = 2 \int \frac{dx}{\sin 2x}$$

$$\begin{cases} 2x = u \\ 2dx = du \end{cases}$$

$$= 2 \int \frac{du}{\sin u} = \int \frac{du}{\sin u} = \ln \tan \frac{u}{2} = \ln(\tan x) + C$$

1126

$$\int \cos \frac{x}{2} \sin \frac{x}{2} dx$$

$$\begin{cases} \frac{x}{2} = u \\ \frac{dx}{2} = du \end{cases}$$

$$= 2 \int \cos u \sin u du =$$

$$\begin{cases} \sin u = z \\ \cos u du = dz \end{cases}$$

$$= 2 \int z dz = z^2 = \sin^2 u = \sin^2 \frac{x}{2} + C$$

1127

$$\int \sin^3 6x \cos 6x dx$$

$$\begin{cases} 6x = t \\ 6dx = dt \end{cases}$$

$$= \int \sin^3 t \cos t \frac{dt}{6} = \frac{1}{6} \int \sin^3 t \cos t dt$$

$$\begin{cases} \sin t = z \\ \cos t dt = dz \end{cases}$$

$$= \frac{1}{6} \int z^3 dz = \frac{1}{24} z^4 = \frac{1}{24} (\sin t)^4 = \frac{1}{24} \sin^4(6x) + C$$

1128

$$\int \frac{\cos ax}{\sin^5 ax} dx$$

$$\begin{cases} ax = t \\ a dx = dt \end{cases}$$

$$= \frac{1}{a} \int \frac{\cos t}{\sin^5 t} dt$$

$$\begin{cases} \sin t = z \\ \cos t dt = dz \end{cases}$$

$$\frac{1}{a} \int \frac{dz}{z^5} = -\frac{1}{4az^4} = -\frac{1}{4a \sin^4 t} = -\frac{1}{4a \sin^4 ax} + C$$

1129

$$\int \frac{\sin 3x}{3+\cos 3x} dx$$

$$\begin{cases} 3 + \cos 3x = z|' \\ -3 \sin 3x dx = dz \end{cases}$$

$$= \int \frac{\sin 3x}{z} \frac{dz}{-3 \sin 3x} = -\frac{1}{3} \int \frac{dz}{z} = -\frac{1}{3} \ln z = -\frac{1}{3} \ln(3 + \cos 3x) + C$$

1130

$$\int \frac{\sin x \cos x}{\sqrt{\cos^2 x - \sin^2 x}} dx = \int \frac{\frac{\sin 2x}{2}}{\sqrt{\cos 2x}} dx = \frac{1}{2} \int \frac{\sin 2x}{\sqrt{\cos 2x}} dx$$

$$\begin{cases} \cos 2x = t|' \\ -2 \sin 2x dx = dt \end{cases}$$

$$= \frac{1}{2} \int \frac{dt}{\sqrt{t}} = -\frac{1}{4} \int \frac{dt}{\sqrt{t}} = -\frac{1}{2} \sqrt{t} = -\frac{1}{2} \sqrt{\cos 2x} + C$$

1131

$$\int \sqrt{1 + 3 \cos^2 x} \sin 2x dx = 2 \int \sin x \cos x \sqrt{1 + 3 \cos^2 x} dx$$

$$\begin{cases} \cos x = t|' \\ -\sin x dx = dt \end{cases}$$

$$= -2 \int t \sqrt{1 + 3t^2} dt$$

$$\begin{cases} 1 + 3t^2 = z \\ 6tdt = dz \end{cases}$$

$$= -\frac{2}{6} \int \sqrt{z} dz = -\frac{2}{9} z^{\frac{3}{2}} = -\frac{2}{9} (1 + 3t^2)^{\frac{3}{2}} = -\frac{2}{9} (1 + 3 \cos^2 x)^{\frac{3}{2}} + C$$

1132

$$\int \tan^3 \frac{x}{3} \sec^2 \frac{x}{3} dx = \int \frac{\sin^3 \frac{x}{3}}{\cos^3 \frac{x}{3}} \frac{1}{\cos^2 \frac{x}{3}} dx = \int \frac{\sin^3 \frac{1}{3} x}{\cos^5 \frac{1}{3} x} dx$$

$$\begin{cases} \cos \frac{x}{3} = t|' \\ -\frac{1}{3} \sin \frac{x}{3} = dt \end{cases}$$

$$= -3 \int \frac{\sin^2 \frac{x}{3} dt}{t^5} = -3 \int \frac{(1-t^2)}{t^5} dt = -3 \int \frac{1}{t^5} dt + 3 \int \frac{1}{t^3} dt =$$

$$3 \frac{1}{4t^4} - 3 \frac{1}{2t^2} = \frac{3}{4 \cos^4 \frac{x}{3}} - \frac{3}{2 \cos^2 \frac{x}{3}} + C$$

1133

$$\begin{aligned} & \int \frac{\sqrt{\tan x}}{\cos^2 x} dx \\ & \left\{ \begin{array}{l} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right\} \\ & = \int \sqrt{t} dt = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3} \tan^{\frac{3}{2}} x + C \end{aligned}$$

1134

$$\begin{aligned} a) \int \frac{\cot^{\frac{2}{3}} x}{\sin^2 x} dx &= \int \frac{\frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{2}{3}} x}}{\sin^2 x} dx = \int \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{8}{3}} x} dx \\ & \left\{ \begin{array}{l} \sin^{\frac{5}{3}} x = t \\ \frac{5}{3} \cos^{\frac{2}{3}} x dx = dt \end{array} \right\} \\ &= \int \frac{\cos^{\frac{2}{3}} x}{\sin^{\frac{8}{3}} x} \frac{dt}{\frac{5}{3} \cos^{\frac{2}{3}} x} = \frac{3}{5} \int \frac{dt}{\sin^{\frac{8}{3}} x} = \frac{3}{5} \int \frac{dt}{\sin^{\frac{8}{3}} x \sin x} = \frac{3}{5} \int \frac{dt}{t \sin x} = \frac{3}{5} \int \frac{dt}{t^{\frac{3}{5}}} = \frac{3}{5} \int \frac{dt}{t^{\frac{8}{5}}} \\ &= \frac{3}{5} \left( -\frac{5}{3t^{\frac{3}{5}}} \right) = -t^{-\frac{3}{5}} = -\left( \sin^{\frac{5}{3}} x \right)^{-\frac{3}{5}} = -\frac{1}{\sin x} + C \\ b) \int \frac{\cot^{\frac{2}{3}} x}{\sin^2 x} dx & \left\{ \begin{array}{l} \cot x = t \\ -\frac{dx}{\sin^2 x} = dt \end{array} \right\} \\ &= -\int t^{\frac{2}{3}} dt = -\frac{3}{5} t^{\frac{5}{3}} = -\frac{3}{5} (\cot x)^{\frac{5}{3}} + C \end{aligned}$$

1135

$$\begin{aligned} \int \frac{1+\sin 3x}{\cos^2 3x} dx &= \int \frac{dx}{\cos^2 3x} + \int \frac{\sin 3x}{\cos^2 3x} dx \\ & \left\{ \begin{array}{l} 3x = t \\ 3dx = dt \end{array} \right\} \left\{ \begin{array}{l} \cos 3x = z \\ -3 \sin 3x dx = dz \end{array} \right\} \\ &= \int \frac{dt}{\cos^2 t} + \int \frac{-dz}{z^2} = \frac{1}{3} \int \frac{dt}{\cos^2 t} - \frac{1}{3} \int z^{-2} dz = \frac{1}{3} \cot t + \frac{1}{3z} = \frac{1}{3} \cot 3x + \frac{1}{3 \cos 3x} + C \end{aligned}$$

1136

$$\begin{aligned} \int \frac{(\cos ax + \sin ax)^2}{\sin ax} dx &= \int \frac{1+2\sin ax \cos ax}{\sin ax} dx = \int \frac{dx}{\sin ax} + 2 \int \cos ax dx \\ & \left\{ \begin{array}{l} ax = t \\ adx = dt \end{array} \right\} \\ &= \frac{1}{a} \int \frac{dt}{\sin t} + \frac{2}{a} \int \cos t dt = \frac{1}{a} \ln |\tan \frac{t}{2}| + \frac{2}{a} \sin t = \frac{1}{a} \ln |\tan \frac{ax}{2}| + \frac{2}{a} \sin ax + C \end{aligned}$$

1137

$$\begin{aligned} \int \frac{\csc^2 3x}{b-a \cot 3x} dx &= \int \frac{\frac{dx}{\sin^2 3x}}{b-a \cot 3x} \\ & \left\{ \begin{array}{l} \cot 3x = t \\ -\frac{3dx}{\sin^2 x} = dt \end{array} \right\} \\ &= \int \frac{dt}{b-at} = -\frac{1}{3} \int \frac{dt}{b-at} = \frac{1}{3a} \ln(b-at) = \frac{1}{3a} \ln(b-a \cot 3x) + C \end{aligned}$$

1138

$$\begin{aligned} \int (2 \sinh 5x - 3 \cosh 5x) dx &= 2 \int \sinh 5x dx - 3 \int \cosh 5x dx \\ & \left\{ \begin{array}{l} 5x = t \\ 5dx = dt \end{array} \right\} \\ &= \frac{2}{5} \int \sinh t dt - \frac{3}{5} \int \cosh t dt = \frac{2}{5} \cosh t - \frac{3}{5} \sinh t = \frac{2}{5} \cosh 5x - \frac{3}{5} \sinh 5x + C \end{aligned}$$

1139

$$\begin{aligned} \int \operatorname{sh}^2 x dx &= \int \left( \frac{e^x - e^{-x}}{2} \right)^2 dx = \int \left( \frac{1}{4} e^{2x} - \frac{1}{2} + \frac{1}{4e^{2x}} \right) dx \\ & \left\{ \begin{array}{l} 2x = t \\ 2dx = dt \end{array} \right\} \\ &= \frac{1}{4} \int e^t \frac{dt}{2} - \frac{1}{2} x + \frac{1}{4} \int \frac{dt}{e^t} = \frac{1}{8} e^t - \frac{1}{2} x - \frac{1}{8e^t} = \frac{1}{8} e^{2x} - \frac{1}{2} x - \frac{1}{8e^{2x}} + C \end{aligned}$$

1140

$$\begin{aligned} \int \frac{dx}{\sinh x} &= \int \frac{2}{e^x - e^{-x}} dx = \int \frac{2}{\frac{e^{2x}-1}{e^x}} dx = \int \frac{2e^x}{e^{2x}-1} dx = \\ & \left\{ \begin{array}{l} e^x = t \\ e^x dx = dt \end{array} \right\} \\ &= \int \frac{2dt}{t^2-1} = -2 \operatorname{arctanh} t = -2 \operatorname{arctanh} e^x + C \end{aligned}$$

1141

$$\begin{aligned} \int \frac{dx}{\cosh x} &= \int \frac{2}{e^x + e^{-x}} dx = 2 \int \frac{dx}{\frac{e^{2x}+1}{e^x}} = 2 \int \frac{e^x dx}{e^{2x}+1} \\ & \left\{ \begin{array}{l} e^x = t \\ e^x dx = dt \end{array} \right\} \\ &= 2 \int \frac{dt}{t^2+1} = 2 \operatorname{arctan} t = 2 \operatorname{arctan} e^x + C \end{aligned}$$

1142

$$\begin{aligned} \int \frac{dx}{\sinh x \cosh x} &= \int \frac{dx}{\sqrt{\frac{1}{2}(\cosh \frac{x}{2}-1)} \sqrt{\frac{1}{2}(\cosh \frac{x}{2}+1)}} = \int \frac{dx}{\sqrt{\frac{1}{4}(\cosh^2 \frac{x}{2}-1)}} = \int \frac{dx}{\frac{1}{2} \sinh \frac{x}{2}} = 2 \int \frac{dx}{\sinh \frac{x}{2}} \\ & \left\{ \begin{array}{l} \frac{x}{2} = u \\ \frac{dx}{2} = du \end{array} \right\} \\ &= \int \frac{du}{\sinh u} = -2 \operatorname{arctanh}(e^u) = -2 \operatorname{arctanh}(e^{\frac{x}{2}}) + C \end{aligned}$$

1143

$$\begin{aligned} \int \tanh x dx &= \int \frac{\sinh x}{\cosh x} dx \\ & \left\{ \begin{array}{l} \cosh x = t \\ \sinh x dx = dt \end{array} \right\} \\ &= \int \frac{dt}{t} = \ln t = \ln \cosh x + C \end{aligned}$$

1144

$$\begin{aligned}\int \coth x dx &= \int \frac{\cosh x}{\sinh x} dx \\ \left\{ \begin{array}{l} \sinh x = t \\ \cosh x dx = dt \end{array} \right\} \\ &= \int \frac{dt}{t} = \ln t = \ln \sinh x + C\end{aligned}$$

1145

$$\begin{aligned}\int x \sqrt{5-x^2} dx \\ \left\{ \begin{array}{l} 5-x^2 = t \\ -2x dx = dt \end{array} \right\} \\ = \int -\sqrt{t} \frac{dt}{2} = -\frac{1}{2} \int \sqrt{t} dt = -\frac{5}{12} t^{\frac{6}{5}} = -\frac{5}{12} (5-x^2)^{\frac{6}{5}} + C\end{aligned}$$

1146

$$\begin{aligned}\int \frac{x^3-1}{x^4-4x+1} dx = \\ \left\{ \begin{array}{l} x^4 - 4x + 1 = t \\ (4x^3 - 4)dx = dt \end{array} \right\} \\ = \int \frac{dt}{t} = \frac{1}{4} \ln t = \frac{1}{4} \ln(x^4 - 4x + 1) + C\end{aligned}$$

1147

$$\begin{aligned}\int \frac{x^3}{x^5+5} dx = \\ \left\{ \begin{array}{l} x^4 = t \\ 4x^3 dx = dt \end{array} \right\} \\ = \int \frac{dt}{t^2+5} = \frac{1}{4} \int \frac{dt}{t^2+5} = \frac{\sqrt{5}}{20} \arctan \frac{t\sqrt{5}}{5} = \frac{\sqrt{5}}{20} \arctan \frac{x^4\sqrt{5}}{5} + C\end{aligned}$$

1148

$$\begin{aligned}\int x e^{-x^2} dx \\ \left\{ \begin{array}{l} -x^2 = u \\ -2x dx = du \end{array} \right\} \\ = \int \frac{e^u}{-2} du = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u = -\frac{1}{2} e^{-x^2} + C\end{aligned}$$

1149

$$\begin{aligned}\int \frac{3-\sqrt{2+3x^2}}{2+3x^2} dx \\ \left\{ \begin{array}{l} 2+3x^2 = t^2 \\ 3x dx = t dt \end{array} \right\} \\ = \int \frac{3-t}{t^2} \frac{tdt}{3x} = \frac{1}{3} \int \frac{3-t}{t^2 \sqrt{\frac{t^2-2}{3}}} t dt = \frac{\sqrt{3}}{3} \int \frac{3-t}{t\sqrt{t^2-2}} dt = \frac{\sqrt{3}}{3} \int \frac{3}{t\sqrt{t^2-2}} - \frac{\sqrt{3}}{3} \int \frac{t}{t\sqrt{t^2-2}} = \sqrt{3} \int \frac{1}{t\sqrt{t^2-2}} \\ \left\{ \begin{array}{l} t^2 - 2 = z^2 \\ t dt = zdz \end{array} \right\}\end{aligned}$$

$$\begin{aligned}&= -\frac{1}{3} \sqrt{3} \ln(t + \sqrt{t^2-2}) + \sqrt{3} \int \frac{1}{tz} \frac{z dz}{t} = -\frac{1}{3} \sqrt{3} \ln(t + \sqrt{(t^2-2)}) + \sqrt{3} \int \frac{1}{(z^2+2)z} zdz \\ &= -\frac{1}{3} \sqrt{3} \ln(t + \sqrt{t^2-2}) + \sqrt{3} \int \frac{1}{z^2+2} dz = -\frac{1}{3} \sqrt{3} \ln(t + \sqrt{t^2-2}) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{\sqrt{2}}{2} \sqrt{t^2-2} \\ &= -\frac{1}{3} \sqrt{3} \ln(\sqrt{2+3x^2} + \sqrt{2+3x^2-2}) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{\sqrt{2}}{2} \sqrt{2+3x^2-2} \\ &= -\frac{1}{3} \sqrt{3} \ln(\sqrt{2+3x^2} + \sqrt{3}x) + \frac{1}{2} \sqrt{3} \sqrt{2} \arctan \frac{1}{2} \sqrt{2} \sqrt{3}x + C\end{aligned}$$

$$\begin{aligned}\int \frac{3-\sqrt{2+3x^2}}{2+3x^2} dx &= \int \frac{3}{2+3x^2} dx - \int \frac{dx}{\sqrt{2+3x^2}} = \int \frac{dx}{x^2+\frac{2}{3}} - \frac{\sqrt{3}}{3} \int \frac{dx}{\sqrt{x^2+\frac{2}{3}}} \\ &= \frac{1}{2} \sqrt{6} \arctan \frac{1}{2} x \sqrt{6} - \left( \frac{\sqrt{3}}{3} \right) \operatorname{arcsinh} \frac{1}{2} x \sqrt{6} = \\ &\quad \frac{1}{2} \sqrt{6} \arctan \frac{1}{2} x \sqrt{6} - \frac{1}{3} \sqrt{3} \operatorname{arcsinh} \frac{1}{2} x \sqrt{6} + C\end{aligned}$$

1150

$$\begin{aligned}\int \frac{(x^3-1)}{(x+1)} dx &= \int x^2 dx - \int x dx + \int dx - 2 \int \frac{dx}{x+1} \\ (x^3-1) : (x+1) &= x^2 - x + 1 - \frac{2}{x+1} \\ x^3+x^2 &\\ -1-x^2 &\\ -x^2-x &\\ -1+x &\\ x+1 &\\ -2 &\\ &\\ &= \frac{1}{3} x^3 - \frac{1}{2} x^2 + x - 2 \ln(x+1) + C\end{aligned}$$

1151

$$\begin{aligned}\int \frac{dx}{\sqrt{e^x}} &= \int e^{-\frac{1}{2}x} dx \\ \left\{ \begin{array}{l} -\frac{1}{2}x = u \\ -\frac{dx}{2} = du \end{array} \right\} \\ &= -2 \int e^u du = -2e^u = -2e^{-\frac{1}{2}x} + C\end{aligned}$$

1152

$$\begin{aligned}\int \frac{1-\sin x}{x+\cos x} dx \\ \left\{ \begin{array}{l} x+\cos x = u \\ (1-\sin x)dx = du \end{array} \right\} \\ &= \int \frac{du}{u} = \ln u = \ln(x+\cos x) + C\end{aligned}$$

1153

$$\begin{aligned}\int \frac{\tan 3x - \cot 3x}{\sin 3x} dx \\ \left\{ \begin{array}{l} 3x = u \\ 3dx = du \end{array} \right\}\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{3} \int \frac{\tan u - \cot u}{\sin u} du = \frac{1}{3} \int \frac{\frac{\sin u}{\cos u} - \frac{\cos u}{\sin u}}{\sin u} du = \frac{1}{3} \int \frac{1}{\cos u} - \frac{1}{3} \int \frac{\cos u}{\sin^2 u} \\
&\left\{ \begin{array}{l} \sin u = t \\ \cos u du = dt \end{array} \right\} \\
&= \frac{1}{3} \ln\left(\frac{1+\sin u}{\cos u}\right) - \frac{1}{3} \int \frac{dt}{t^2} = \frac{1}{3} \ln \frac{1+\sin u}{\cos u} + \frac{1}{3t} = \frac{1}{3} \ln \frac{1+\sin u}{\cos u} + \frac{1}{3 \sin u} = \\
&\frac{1}{3} \ln \frac{1+\sin 3x}{\cos 3x} + \frac{1}{3 \sin 3x} + C
\end{aligned}$$

1154

$$\begin{aligned}
&\int \frac{dx}{x \ln^2 x} \\
&\left\{ \begin{array}{l} \ln x = t \\ \frac{dx}{x} = dt \end{array} \right\} \\
&= \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{\ln x} + C
\end{aligned}$$

1155

$$\begin{aligned}
&\int \frac{\sec^2 x}{\sqrt{\tan^2 x - 2}} dx = \int \frac{\frac{dx}{\cos^2 x}}{\sqrt{\tan^2 x - 2}} \\
&\left\{ \begin{array}{l} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right\} \\
&= \int \frac{dt}{\sqrt{t^2 - 2}} = \ln\left(t + \sqrt{(t^2 - 2)}\right) = \ln\left(\tan x + \sqrt{(\tan^2 x - 2)}\right) + C
\end{aligned}$$

1156

$$\begin{aligned}
&\int \left(2 + \frac{x}{2x^2+1}\right) \frac{dx}{2x^2+1} = \int \frac{2}{2x^2+1} dx + \int \frac{x}{(2x^2+1)^2} dx \\
&\left\{ \begin{array}{l} 2x^2 + 1 = t \\ 4x dx = dt \end{array} \right\} \\
&= \int \frac{dx}{x^2 + \frac{1}{2}} + \int \frac{dt}{4t^2} = \sqrt{2} \arctan x \sqrt{2} - \frac{1}{4t} = \sqrt{2} \arctan x \sqrt{2} - \frac{1}{4(2x^2+1)} + C
\end{aligned}$$

1157

$$\begin{aligned}
&\int a^{\sin x} \cos x dx \\
&\left\{ \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right\} \\
&= \int a^t dt = \frac{1}{\ln a} a^t = \frac{1}{\ln a} a^{\sin x} + C
\end{aligned}$$

1158

$$\begin{aligned}
&\int \frac{x^2}{\sqrt[3]{x^3+1}} dx \\
&\left\{ \begin{array}{l} x^3 + 1 = u' \\ 3x^2 dx = du \end{array} \right\} \\
&= \int \frac{du}{3\sqrt[3]{u}} = \frac{1}{2} \sqrt[3]{u^2} = \frac{1}{2} \sqrt[3]{(x^3+1)^2} + C
\end{aligned}$$

1159

$$\begin{aligned}
&\int \frac{xdx}{\sqrt{1-x^4}} \\
&\left\{ \begin{array}{l} x^2 = u' \\ 2x dx = du \end{array} \right\} \\
&= \int \frac{du}{2\sqrt{1-u^2}} = \frac{1}{2} \arcsin u = \frac{1}{2} \arcsin x^2 + C
\end{aligned}$$

1160

$$\begin{aligned}
&\int \tan^2 ax dx \\
&\left\{ \begin{array}{l} ax = u' \\ adx = du \end{array} \right\} \\
&= \frac{1}{a} \int \tan^2 u du = \frac{1}{a} \int \frac{\sin^2 u}{\cos^2 u} du = \frac{1}{a} \int \frac{1-\cos^2 u}{\cos^2 u} du = \frac{1}{a} \int \frac{du}{\cos^2 u} - \frac{1}{a} \int du = \frac{1}{a} \tan u - \frac{u}{2} = \\
&\frac{1}{a} \tan ax - \frac{1}{2} ax + C
\end{aligned}$$

1161

$$\begin{aligned}
&\int \sin^2 \frac{x}{2} dx \\
&\left\{ \begin{array}{l} \frac{x}{2} = t \\ \frac{dx}{2} = dt \end{array} \right\} \\
&= 2 \int \sin^2 t dt = 2 \int \frac{1}{2} (1 - \cos 2t) = \int (1 - \cos 2t) = t - \frac{1}{2} \sin 2t = \frac{x}{2} - \frac{1}{2} \sin x + C
\end{aligned}$$

1162

$$\begin{aligned}
&\int \frac{\sec^2 x dx}{\sqrt{4-\tan^2 x}} \\
&\left\{ \begin{array}{l} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right\} \\
&= \int \frac{dt}{\sqrt{4-t^2}} = \arcsin \frac{t}{2} = \arcsin \frac{\tan x}{2} + C
\end{aligned}$$

1163

$$\begin{aligned}
&\int \frac{dx}{\cos \frac{x}{a}} \\
&\left\{ \begin{array}{l} \frac{x}{a} = t \\ \frac{dx}{a} = dt \end{array} \right\} \\
&= a \int \frac{dt}{\cos t} = a \ln(\sec t + \tan t) = a \ln\left(\frac{1+\sin \frac{x}{a}}{\cos \frac{x}{a}}\right) + C
\end{aligned}$$

1164

$$\begin{aligned}
&\int \frac{\sqrt[3]{1+\ln x}}{x} dx \\
&\left\{ \begin{array}{l} 1 + \ln x = t \\ \frac{dx}{x} = dt \end{array} \right\} \\
&= \int \sqrt[3]{t} dt = \frac{3}{4} \sqrt[3]{t^4} = \frac{3}{4} \sqrt[3]{(1+\ln x)^4} + C
\end{aligned}$$

1165

$$\int \tan \sqrt{x-1} \frac{dx}{\sqrt{x-1}}$$

$$\left\{ \begin{array}{l} \sqrt{x-1} = t \\ \frac{dx}{2\sqrt{x-1}} = dt \end{array} \right\}$$

$$= 2 \int \tan t dt = -2 \ln(\cos t) = -2 \ln(\cos \sqrt{x-1}) + C$$

1166

$$\int \frac{xdx}{\sin x^2}$$

$$\left\{ \begin{array}{l} x^2 = t \\ 2xdx = dt \end{array} \right\}$$

$$= \frac{1}{2} \int \frac{dt}{\sin t} = \frac{1}{2} \ln\left(\frac{1-\cos t}{\sin t}\right) = \frac{1}{2} \ln\left(\frac{1-\cos x^2}{\sin x^2}\right) + C$$

1167

$$\int \frac{e^{\arctan x+x \ln(1+x^2)+1}}{1+x^2} dx = \int \frac{e^{\arctan x}}{1+x^2} dx + \int \frac{x \ln(1+x^2)}{1+x^2} dx + \int \frac{dx}{1+x^2}$$

$$\left\{ \begin{array}{l} \arctan x = u \\ \frac{dx}{1+x^2} = du \end{array} \right\} \left\{ \begin{array}{l} \ln(1+x^2) = t \\ \frac{2xdx}{1+x^2} = dt \end{array} \right\}$$

$$= \int e^u du + \frac{1}{2} \int t dt + \arctan x = e^u + \frac{1}{4} t^2 + \arctan x =$$

$$e^{\arctan x} + \frac{1}{4} \ln^2(1+x^2) + \arctan x + C$$

1168

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx =$$

$$\left\{ \begin{array}{l} \sin x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ dx = \frac{2dt}{1+t^2} \end{array} \right\}$$

$$\int \frac{\frac{2t}{1+t^2} - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \frac{2dt}{1+t^2} = 2 \int \frac{t^2+2t-1}{(2t+1-t^2)(1+t^2)} dt =$$

$$\frac{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}}{(2t+1-t^2)(1+t^2)} = \frac{at+b}{(2t+1-t^2)} + \frac{ct+d}{(1+t^2)}$$

$$t^2 + 2t - 1 = (at+b)(1+t^2) + (ct+d)(2t+1-t^2)$$

$$t^2 + 2t - 1 = at + at^3 + b + bt^2 + 2ct^2 + ct - ct^3 + 2dt + d - dt^2$$

$$t^2 + 2t - 1 = t^3(a-c) + t^2(b+2c-d) + t(a+c+2d) + b + d$$

$$a - c = 0$$

$$b + 2c - d = 1$$

$$a + c + 2d = 2$$

$$b + d = -1$$

$$\{a = 1, d = 0, c = 1, b = -1\}$$

$$\int \frac{t^2+2t-1}{(2t+1-t^2)(1+t^2)} dt = \int \frac{t-1}{(2t+1-t^2)} dt + \int \frac{t}{(1+t^2)} dt$$

$$\left\{ \begin{array}{l} 2t+1-t^2 = z \\ 2(1-t)dt = dz \end{array} \right\} \left\{ \begin{array}{l} 1+t^2 = u \\ 2tdt = du \end{array} \right\}$$

$$= 2 \left( -\int \frac{dt}{2z} + \int \frac{du}{2u} \right) = -\frac{1}{2} \ln z + \frac{1}{2} \ln u$$

$$= -\ln(2t+1-t^2) + \ln(1+t^2) = -\ln(2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}) + \ln(1 + \tan^2 \frac{x}{2}) + C$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{\sin x - \cos x}{\sin x + \cos x} \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{\sin^2 x - 2 \sin x \cos x + \cos^2 x}{\sin^2 x + \cos^2 x} dx$$

$$= \int \frac{1-2\sin x \cos x}{\sin^2 x + \cos^2 x} dx = \int \frac{1-\sin 2x}{-\cos 2x} dx = -\int \frac{1-\sin 2x}{\cos 2x} dx = -\int \frac{dx}{\cos 2x} + \int \tan 2x dx$$

$$\left\{ \begin{array}{l} 2x = t \\ 2dx = dt \end{array} \right\}$$

$$= -\frac{1}{2} \int \frac{dt}{\cos t} + \frac{1}{2} \int \tan t dt = -\frac{1}{2} \ln\left(\frac{1+\sin t}{\cos t}\right) - \frac{1}{2} \ln(\cos t) =$$

$$-\frac{1}{2} \ln\left(\frac{1+\sin 2x}{\cos 2x}\right) - \frac{1}{2} \ln(\cos 2x) + C$$

1169

$$\int \frac{\left(1-\sin \frac{x}{\sqrt{2}}\right)^2}{\sin \frac{x}{\sqrt{2}}} dx$$

$$\left\{ \begin{array}{l} \frac{x}{\sqrt{2}} = t \\ \frac{dx}{\sqrt{2}} = dt \end{array} \right\}$$

$$= \sqrt{2} \int \frac{(1-\sin t)^2}{\sin t} = \sqrt{2} \int \frac{(1-\sin t)^2}{\sin t} = \sqrt{2} \int \frac{1-2\sin t + \sin^2 t}{\sin t} dt = \sqrt{2} \left( \int \frac{dt}{\sin t} - 2 \int dt + \int \sin t \right)$$

$$= \sqrt{2} \left( \ln\left(\frac{1-\cos t}{\sin t}\right) - 2t - \cos t \right) = \sqrt{2} \left( \ln\left(\frac{1-\cos \frac{x}{\sqrt{2}}}{\sin \frac{x}{\sqrt{2}}}\right) - 2 \frac{x}{\sqrt{2}} - \cos \frac{x}{\sqrt{2}} \right) + C$$

1170

$$\int \frac{x^2}{x^2-2} dx = \int \frac{x^2-2+2}{x^2-2} dx = \int dx + 2 \int \frac{dx}{x^2-2} = x - \sqrt{2} \operatorname{arctanh} \frac{1}{2} x \sqrt{2} + C$$

1171

$$\int \frac{(1+x)^2}{x(1+x^2)} dx = \int \frac{x^2+2x+1}{x(1+x^2)} dx = \int \frac{x}{x(1+x^2)} dx + 2 \int \frac{dx}{(1+x^2)} + \int \frac{dx}{x(1+x^2)}$$

$$= \int \frac{x}{(1+x^2)} dx + 2 \operatorname{arctan} x + \int \frac{dx}{x(1+x^2)}$$

$$\left\{ \begin{array}{l} 1+x^2 = t \\ 2xdx = dt \end{array} \right\}$$

$$= \int \frac{dt}{t} + 2 \operatorname{arctan} x + \int \frac{dx}{x(1+x^2)} = \frac{1}{2} \ln t + 2 \operatorname{arctan} x + \int \frac{dx}{x(1+x^2)}$$

$$\frac{1}{x(1+x^2)} = \frac{a}{x} + \frac{bx+c}{(1+x^2)}$$

$$1 = a(1+x^2) + bx(x+c) = a + ax^2 + bx^2 + xc$$

$$1 = x^2(a+b) + x(c+a)$$

$$a+b=0 \rightarrow$$

$$\{a=1\}, \{b=-1\}, \{c=0\}$$

$$I_1 = \int \frac{a}{x} dx + \int \frac{bx+c}{(1+x^2)} dx = a \ln x + \int \frac{-x}{(1+x^2)} dx = \ln x - \frac{1}{2} \ln(1+x^2)$$

$$I = \frac{1}{2} \ln(1+x^2) + 2 \operatorname{arctan} x + \ln x - \frac{1}{2} \ln(1+x^2) = 2 \operatorname{arctan} x + \ln x + C$$

1172

$$\int e^{\sin^2 x} \sin 2x dx = 2 \int e^{\sin^2 x} \sin x \cos x dx$$

$$\left\{ \begin{array}{l} \sin^2 x = t \\ 2 \sin x \cos x dx = dt \end{array} \right\}$$

$$= \int e^t dt = e^{\sin^2 x} + C$$

1173

$$\int \frac{-5-3x}{\sqrt{4-3x^2}} dx = \int \frac{5}{\sqrt{4-3x^2}} dx - \int \frac{3x}{\sqrt{4-3x^2}} dx = \int \frac{5}{\sqrt{4-3x^2}} : \sqrt{3} dx - \int \frac{3x}{\sqrt{4-3x^2}} dx$$

$$\left\{ \begin{array}{l} 4-3x^2 = t \\ -6xdx = dt \end{array} \right\}$$

$$= \frac{5}{\sqrt{3}} \int \frac{dx}{\sqrt{\frac{4}{3}-x^2}} - \int \frac{3 \cdot \frac{dt}{-6}}{\sqrt{t}} = \frac{5}{\sqrt{3}} \arcsin \frac{1}{2} \sqrt{3} x + \frac{1}{2} \int \frac{dt}{\sqrt{t}}$$

$$= \frac{5}{3} \sqrt{3} \arcsin \frac{1}{2} \sqrt{3} x + \sqrt{t} = \frac{5}{3} \sqrt{3} \arcsin \frac{1}{2} \sqrt{3} x + \sqrt{4-3x^2} + C$$

1174

$$\int \frac{dx}{e^x+1}$$

$$\left\{ \begin{array}{l} e^x + 1 = t \\ e^x dx = dt \end{array} \right\}$$

$$= \int \frac{dt}{e^x t} = \int \frac{dt}{(t-1)t} = \int \frac{dt}{t^2-t} = \int \frac{dt}{(t-\frac{1}{2})^2-\frac{1}{4}}$$

$$\left\{ \begin{array}{l} t-\frac{1}{2} = u \\ dt = du \end{array} \right\}$$

$$= \int \frac{du}{u^2-\frac{1}{4}} = \frac{1}{2\sqrt{\frac{1}{4}}} \ln \frac{u-\frac{1}{2}}{u+\frac{1}{2}} = \ln \frac{u-\frac{1}{2}}{u+\frac{1}{2}} = \ln(2u-1) - \ln(2u+1)$$

$$= \ln(2(t-\frac{1}{2})-1) - \ln(2(t-\frac{1}{2})+1) = \ln(2t-2) - \ln 2t$$

$$= \ln(2(e^x+1)-2) - \ln 2(e^x+1) + C$$

1175

$$\int \frac{dx}{(a+b)+(a+b)x^2} = \frac{1}{(a+b)} \int \frac{dx}{1+x^2} = \frac{1}{a+b} \arctan x + C$$

1176

$$\int \frac{e^x}{\sqrt{e^{2x}-1}} dx$$

$$\left\{ \begin{array}{l} e^x = t \\ e^x dx = dt \end{array} \right\}$$

$$= \int \frac{dt}{\sqrt{t^2-1}} = \ln(t + \sqrt{t^2-1}) = \ln(e^x + \sqrt{e^{2x}-1}) + C$$

1177

$$\int \frac{dx}{\sin ax \cos ax} = \int \frac{dx}{\frac{\sin 2ax}{2}} = 2 \int \frac{dx}{\sin 2ax}$$

$$\left\{ \begin{array}{l} 2ax = t \\ 2adx = dt \end{array} \right\}$$

$$= 2 \int \frac{\frac{dt}{2a}}{\sin t} = \frac{1}{a} \int \frac{dt}{\sin t} = \frac{1}{a} \ln \frac{1-\cos t}{\sin t} = \frac{1}{a} \ln \frac{1-\cos 2ax}{\sin 2ax} + C$$

1178

$$\int \sin(\frac{2\pi t}{T} + \varphi_0) dt = \int \sin(\frac{2\pi t}{T} + \varphi_0) dt =$$

$$\left\{ \begin{array}{l} \frac{2\pi t}{T} + \varphi_0 = x \\ \frac{2\pi}{T} dt = dx \end{array} \right\}$$

$$= \frac{T}{2\pi} \int \sin x dx = -\frac{1}{2} \frac{T}{\pi} \cos x = -\frac{T}{2\pi} \cos(\frac{2\pi t}{T} + \varphi_0) + C$$

1179

$$\int \frac{dx}{x(4-\ln^2 x)}$$

$$\left\{ \begin{array}{l} \ln x = t \\ \frac{dx}{x} = dt \end{array} \right\}$$

$$= \int \frac{x dt}{x(4-t^2)} = \int \frac{dt}{4-t^2} = \frac{1}{4} \ln \frac{2+t}{2-t} = \frac{1}{4} \ln \frac{\ln x + 2}{2 - \ln x} + C$$

1180

$$\int \frac{\arccos \frac{x}{2}}{\sqrt{1-x^2}} dx$$

$$\left\{ \begin{array}{l} \arccos \frac{x}{2} = t \\ -\frac{1}{\sqrt{1-(\frac{x}{2})^2}} dx = -\frac{1}{\sqrt{(4-x^2)}} dx = dt \end{array} \right\}$$

$$= -\int t dt = -\frac{1}{2} t^2 = -\frac{1}{2} \arccos^2 \frac{x}{2} + C$$

1181

$$\int e^{-\tan x} \sec^2 x dx = \int e^{-\tan x} \frac{1}{\cos^2 x} dx$$

$$\left\{ \begin{array}{l} -\tan x = u \\ -\frac{1}{\cos^2 x} dx = du \end{array} \right\}$$

$$= -\int e^u du = -e^u = -e^{-\tan x} + C$$

1182

$$\int \frac{\sin x \cos x}{\sqrt{2-\sin^4 x}} dx$$

$$\left\{ \begin{array}{l} \sin^2 x = t \\ 2 \sin x \cos x dx = dt \end{array} \right\}$$

$$= \int \frac{dt}{2\sqrt{2-t^2}} = \frac{1}{2} \int \frac{dt}{\sqrt{2-t^2}} = \frac{1}{2} \arcsin \frac{\sqrt{2}}{2} t = \frac{1}{2} \arcsin \left( \frac{\sqrt{2}}{2} \sin^2 x \right) + C$$

1183

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \int \frac{dx}{\frac{\sin^2 2x}{4}} = 4 \int \frac{dx}{\sin^2 2x}$$

$$\left\{ \begin{array}{l} 2x = u \\ 2dx = du \end{array} \right\}$$

$$= 4 \int \frac{\frac{du}{2}}{\sin^2 u} = 2 \int \frac{du}{\sin^2 u} = -2 \cot u = -2 \cot 2x + C$$

1184

$$\begin{aligned} \int \frac{\arcsin x+x}{\sqrt{1-x^2}} dx &= \int \frac{\arcsin x}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}} \\ \left\{ \begin{array}{l} \arcsin x = u \\ \frac{dx}{\sqrt{1-x^2}} = du \end{array} \right. &\left\{ \begin{array}{l} 1-x^2 = t \\ -2xdx = dt \end{array} \right. \\ &= \int u du - \frac{1}{2} \int \frac{dt}{\sqrt{t}} = \frac{1}{2} u^2 - \sqrt{t} = \frac{1}{2} \arcsin^2 x - \sqrt{1-x^2} + C \end{aligned}$$

1185

$$\begin{aligned} \int \frac{\sec x \tan x}{\sqrt{\sec^2 x + 1}} dx &= \int \frac{\frac{1}{\cos x} \frac{\sin x}{\cos^2 x}}{\sqrt{\frac{1+\cos^2 x}{\cos^2 x}}} dx = \int \frac{\frac{\sin x}{\cos^2 x}}{\sqrt{\frac{1+\cos^2 x}{\cos^2 x}}} dx = \int \frac{\sin x}{\cos x \sqrt{1+\cos^2 x}} dx \\ \left\{ \begin{array}{l} \sqrt{1+\cos^2 x} = t^2 \\ 1+\cos^2 x = t^2 \\ 2 \sin x \cos x dx = 2tdt \end{array} \right. & \\ &= \int \frac{\sin x}{\cos x t} \frac{tdt}{\sin x \cos x} = \int \frac{dt}{\cos^2 x} = \int \frac{dt}{t^2-1} = -\operatorname{arctanh} t = -\operatorname{arctanh} \sqrt{1+\cos^2 x} + C \end{aligned}$$

1186

$$\begin{aligned} \int \frac{\cos 2x}{4+\cos^2 2x} dx & \\ \left\{ \begin{array}{l} 2x = t \\ 2dx = dt \end{array} \right. & \\ &= \frac{1}{2} \int \frac{\cos t}{4+\cos^2 t} = \frac{1}{2} \int \frac{\cos t}{5-1+\cos^2 t} = \frac{1}{2} \int \frac{\cos t}{5-\sin^2 t} \\ \left\{ \begin{array}{l} \sin t = u \\ \cos t dt = du \end{array} \right. & \\ &= \frac{1}{2} \int \frac{du}{5-u^2} = \frac{\sqrt{5}}{10} \operatorname{arctanh} \frac{\sqrt{5}}{5} u = \frac{\sqrt{5}}{10} \operatorname{arctanh} \left( \frac{\sqrt{5}}{5} \sin t \right) = \\ &\frac{\sqrt{5}}{10} \operatorname{arctanh} \left( \frac{\sqrt{5}}{5} \sin 2x \right) + C \end{aligned}$$

1187\*

$$\begin{aligned} \int \frac{dx}{1+\cos^2 x} &= \int \frac{dx}{\sin^2 x + \cos^2 x + \cos^2 x} \cdot \cos^2 x = \int \frac{\frac{dx}{\cos^2 x}}{\tan^2 x + 2} = \int \frac{1}{\cos^2 x (\tan^2 x + 2)} dx \\ \left\{ \begin{array}{l} \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right. & \\ &= \int \frac{dt}{t^2+2} = \frac{\sqrt{2}}{2} \arctan \frac{t\sqrt{2}}{2} = \frac{\sqrt{2}}{2} \arctan \frac{\tan x \sqrt{2}}{2} + C \end{aligned}$$

1188

$$\int \sqrt{\frac{\ln(x+\sqrt{x^2+1})}{x^2+1}} dx$$

$$\begin{aligned} &\left\{ \begin{array}{l} \ln(x+\sqrt{x^2+1}) = t \\ \frac{1}{(x+\sqrt{x^2+1})} \left( 1 + \frac{2x}{2\sqrt{x^2+1}} \right) dx = dt \\ \frac{1}{(x+\sqrt{x^2+1})} \left( \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}} \right) dx = dt \\ \frac{1}{\sqrt{x^2+1}} dx = dt \end{array} \right. \\ &= \int \sqrt{t} dt = \frac{2}{3} t^{\frac{3}{2}} = \frac{2}{3} \ln^{\frac{3}{2}} \left( x + \sqrt{x^2+1} \right) + C \end{aligned}$$

1189

$$\begin{aligned} &\int x^2 ch(x^3 + 3) dx \\ \left\{ \begin{array}{l} x^3 + 3 = t \\ 3x^2 dx = dt \end{array} \right. & \\ &= \frac{1}{3} \int \cosh t dt = \frac{1}{3} \sinh t = \frac{1}{3} \sinh(x^3 + 3) + C \end{aligned}$$

1190

$$\begin{aligned} &\int \frac{3^{\tanh x}}{\cosh^2 x} dx \\ \left\{ \begin{array}{l} \tanh x = t \\ \frac{dx}{\cosh^2 x} = dt \end{array} \right. & \\ &= \int 3^t dt = \frac{1}{\ln 3} 3^t = \frac{1}{\ln 3} 3^{\tanh x} + C \end{aligned}$$

1192

$$\begin{aligned} &\int x(2x+5)^{10} dx \\ \left\{ \begin{array}{l} 2x+5 = t \\ 2dx = dt \end{array} \right. & \\ &= \frac{1}{2} \int t^{10} dt = \frac{1}{22} t^{11} = \frac{1}{22} (2x+5)^{11} + C \end{aligned}$$

1191a)

$$\begin{aligned} &\int \frac{dx}{x\sqrt{x^2-2}} \\ \left\{ \begin{array}{l} \sqrt{x^2-2} = t \\ \frac{xdx}{\sqrt{x^2-2}} = dt \end{array} \right. & \\ &= \int \frac{dt}{x\sqrt{x^2-2}} = \int \frac{dt}{x^2} = \int \frac{dt}{t^2+2} = \frac{\sqrt{2}}{2} \arctan \frac{\sqrt{2}}{2} t \\ &= \frac{\sqrt{2}}{2} \arctan \frac{\sqrt{2}}{2} \sqrt{x^2-2} + C \\ &\int \frac{dx}{x\sqrt{x^2-2}} \\ \left\{ \begin{array}{l} \frac{1}{x} = t \\ -\frac{dx}{x^2} = dt \end{array} \right. & \end{aligned}$$

$$= \int \frac{-x^2 dt}{x\sqrt{(\frac{1}{t})^2 - 2}} = - \int \frac{\frac{1}{t} dt}{\sqrt{\frac{2-x^2}{t^2}}} = - \int \frac{dt}{\sqrt{(2-t^2)}} = - \arcsin \frac{\sqrt{2}}{2} t = - \arcsin \frac{\sqrt{2}}{2x} + C$$

1191b

$$\begin{aligned} & \int \frac{dx}{e^x + 1} \\ & \left\{ \begin{array}{l} x = -\ln t \\ dx = -\frac{dt}{t} \end{array} \right\} \\ & = \int \frac{-\frac{dt}{t}}{e^{-\ln t} + 1} = - \int \frac{\frac{dt}{t}}{\frac{1}{t} + 1} = - \int \frac{dt}{1+t} = - \ln(1+t) = - \ln(1 + \frac{1}{e^x}) + C \end{aligned}$$

1191c

$$\begin{aligned} & \int x(5x^2 - 3)^7 dx \\ & \left\{ \begin{array}{l} 5x^2 - 3 = t \\ 10x dx = dt \end{array} \right\} \\ & = \int x(5x^2 - 3)^7 \frac{dt}{10x} = \frac{1}{10} \int t^7 dt = \frac{1}{80} t^8 = \frac{1}{80} (5x^2 - 3)^8 + C \end{aligned}$$

1191d

$$\begin{aligned} & \int \frac{x dx}{\sqrt{x+1}} \\ & \left\{ \begin{array}{l} \sqrt{x+1} = t \\ \frac{dx}{2\sqrt{x+1}} = dt \end{array} \right\} \\ & = 2 \int x dt = 2 \int (t^2 - 1) dt = \frac{2}{3} t^3 - 2t = \frac{2}{3} \sqrt{x+1}^3 - 2\sqrt{x+1} + C \end{aligned}$$

1191e

$$\begin{aligned} & \int \frac{\cos x dx}{\sqrt{1+\sin^2 x}} \\ & \left\{ \begin{array}{l} t = \sin x \\ dt = \cos x dx \end{array} \right\} \\ & = \int \frac{dt}{\sqrt{1+t^2}} = \operatorname{arcsinh} t = \operatorname{arcsinh} \sin x + C \end{aligned}$$

1192

$$\begin{aligned} & \int x(2x-5)^{10} dx \\ & \left\{ \begin{array}{l} 2x-5 = t \quad x = \frac{t+5}{2} \\ 2dx = dt \end{array} \right\} \\ & = \int \frac{t+5}{2} t^{10} \frac{dt}{2} = \frac{1}{4} \int (t+5)t^{10} dt = \frac{1}{48} t^{12} + \frac{5}{44} t^{11} = \frac{1}{48} (2x-5)^{12} + \frac{5}{44} (2x-5)^{11} + C \end{aligned}$$

1193

$$\begin{aligned} & \int \frac{1+x}{1+\sqrt{x}} dx \\ & \left\{ \begin{array}{l} \sqrt{x} = t \\ \frac{dx}{2\sqrt{x}} = dt \end{array} \right\} \end{aligned}$$

$$\begin{aligned} & = \int \frac{1+x}{1+\sqrt{x}} 2\sqrt{x} dt = 2 \int \frac{1+t^2}{1+t} t dt = 2 \int \frac{t+t^3}{1+t} dt \\ & (t^3 + t) : (t+1) = t^2 - t + 2 - \frac{2}{(t+1)} \end{aligned}$$

$$t^3 + t^2$$

$$-t^2 + t$$

$$-t^2 - t$$

$$2t$$

$$2t + 2$$

$$-2$$

$$\begin{aligned} & = 2 \left( \int t^2 dt - \int t dt + 2 \int dt - 2 \int \frac{dt}{t+1} \right) = \frac{2}{3} t^3 - t^2 + 4t - 4 \ln(1+t) \\ & = \frac{2}{3} \sqrt{x}^3 - \sqrt{x}^2 + 4\sqrt{x} - 4 \ln(1+\sqrt{x}) + C \end{aligned}$$

1194

$$\begin{aligned} & \int \frac{dx}{x\sqrt{2x+1}} \\ & \left\{ \begin{array}{l} \sqrt{2x+1} = t \quad 2x+1 = t^2 \\ \frac{dx}{\sqrt{2x+1}} = dt \quad x = \frac{t^2-1}{2} \end{array} \right\} \\ & = \int \frac{\sqrt{2x+1}}{x\sqrt{2x+1}} dt = \int \frac{dt}{x} = \int \frac{dt}{\frac{t^2-1}{2}} = 2 \int \frac{dt}{t^2-1} = -2 \operatorname{arctanh} t = -2 \operatorname{arctanh} \sqrt{2x+1} + C \end{aligned}$$

1195

$$\begin{aligned} & \int \frac{dx}{\sqrt{e^x - 1}} \\ & \left\{ \begin{array}{l} \sqrt{e^x - 1} = t \\ \frac{e^x dx}{2\sqrt{e^x - 1}} = dt \end{array} \right\} \\ & = \int \frac{\frac{e^x}{\sqrt{e^x - 1}} dt}{\sqrt{e^x - 1}} = 2 \int \frac{dt}{e^x} = 2 \int \frac{dt}{t^2 + 1} = 2 \operatorname{arctan} t = 2 \operatorname{arctan} \sqrt{e^x - 1} + C \end{aligned}$$

1196

$$\begin{aligned} & \int \frac{\ln 2x dx}{x \ln 4x} = \int \frac{\ln 2x dx}{x(\ln 2x + \ln 2)} \\ & \left\{ \begin{array}{l} \ln 2x = t \\ \frac{dx}{x} = dt \end{array} \right\} \\ & = \int \frac{tx dt}{x(t+\ln 2)} = \int \frac{tdt}{(t+\ln 2)} = \int \frac{t+\ln 2 - \ln 2}{(t+\ln 2)} dt \\ & = \int dt - \ln 2 \int \frac{dt}{(t+\ln 2)} = t - \ln 2 \ln(t + \ln 2) = \ln 2x - \ln 2 \ln(\ln 2x + \ln 2) + C \end{aligned}$$

1197

$$\begin{aligned} & \int \frac{(\arcsin x)^2}{\sqrt{1-x^2}} dx \\ & \left\{ \begin{array}{l} \arcsin x = t \\ \frac{dx}{\sqrt{1-x^2}} = dt \end{array} \right\} \\ & = \int t^2 dt = \frac{1}{3} t^3 = \frac{1}{3} \arcsin^3 x + C \end{aligned}$$

1198

$$\int \frac{e^{2x}}{\sqrt{e^x+1}} dx$$

$$\left\{ \begin{array}{l} \sqrt{e^x+1} = t \\ \frac{e^x dx}{2\sqrt{e^x+1}} = dt \end{array} \right\}$$

$$= \int \frac{e^{2x}}{t} \frac{2\sqrt{e^x+1}}{e^x} dt = 2 \int \frac{(t^2-1)t}{t} dt = 2 \int (t^2-1)dt = \frac{2}{3}t^3 - 2t =$$

$$\frac{2}{3}(\sqrt{e^x+1})^3 - 2\sqrt{e^x+1} + C$$

1199

$$\int \frac{\sin^3 x}{\cos x} dx$$

$$\left\{ \begin{array}{l} \sqrt{\cos x} = t \\ -\frac{\sin x}{2\sqrt{\cos x}} dx = dt \end{array} \right\}$$

$$= -2 \int \sin^2 x dt = -2 \int (1 - \cos^2 x) dt = -2 \int (1 - t^4) dt = \frac{2}{5}t^5 - 2t =$$

$$\frac{2}{5}\sqrt{\cos x^5} - 2\sqrt{\cos x} + C$$

1200\*

$$\int \frac{dx}{x\sqrt{1+x^2}}$$

$$\left\{ \begin{array}{l} \sqrt{1+x^2} = t^2 \\ 1+x^2 = t^2 \\ xdx = tdt \end{array} \right\}$$

$$= \int \frac{dt}{xt} = \int \frac{1}{x^2} dt = \int \frac{1}{t^2-1} dt = -\operatorname{arctanh} t = -\operatorname{arctanh} \sqrt{1+x^2} + C$$

1201

$$\int \frac{x^2}{\sqrt{1-x^2}} dx$$

$$\left\{ \begin{array}{l} x = \cos t \\ dx = -\sin t dt \end{array} \right\}$$

$$= -\int \frac{\cos^2 t}{\sqrt{1-\cos^2 t}} \sin t dt = -\int \frac{\cos^2 t \sin t}{\sin t} dt = -\int \cos^2 t dt = -\int \frac{1+\cos 2t}{2} dt = -\frac{1}{2}t - \frac{1}{4}\sin 2t$$

$$= -\frac{1}{2}\arccos x - \frac{1}{4}\sin(2\arccos x) + C$$

1202

$$\int \frac{x^3}{\sqrt{2-x^2}} dx$$

$$\left\{ \begin{array}{l} 2-x^2 = \cos t \\ -2xdx = -\sin t dt \end{array} \right\}$$

$$= \int \frac{x^3}{\sqrt{\cos t}} \frac{\sin t dt}{2x} = \int \frac{x^2}{\sqrt{\cos t}} \frac{\sin t dt}{2} = \int \frac{2-\cos t}{\sqrt{\cos t}} \frac{\sin t dt}{2} = \int \frac{2\sin t - \cos t \sin t}{2\sqrt{\cos t}} dt$$

$$= \int \frac{\sin t}{\sqrt{\cos t}} dt - \frac{1}{2} \int \sqrt{\cos t} \sin t dt$$

$$\left\{ \begin{array}{l} \cos t = u \\ -\sin t dt = du \end{array} \right\}$$

$$= -\int \frac{du}{\sqrt{u}} + \frac{1}{2} \int \sqrt{u} du = -2\sqrt{u} + \frac{1}{3}u^{\frac{3}{2}} = -2\sqrt{\cos t} + \frac{1}{3}(\cos t)^{\frac{3}{2}}$$

$$= -2\sqrt{2-x^2} + \frac{1}{3}(2-x^2)^{\frac{3}{2}} + C$$

1203

$$\int \frac{\sqrt{x^2-a^2}}{x} dx$$

$$\left\{ \begin{array}{l} x^2 - a^2 = \sin^2 t \\ xdx = \sin t \cos t dt \end{array} \right\}$$

$$= \int \frac{\sin t}{x} \frac{\sin t \cos t dt}{x} = \int \frac{\sin^2 t \cos t dt}{x^2} = \int \frac{\sin^2 t \cos t dt}{\sin^2 t + a^2}$$

$$\left\{ \begin{array}{l} \sin t = z \\ \cos t dt = dz \end{array} \right\}$$

$$= \int \frac{z^2 dz}{z^2 + a^2} = \int dz - a^2 \int \frac{dz}{z^2 + a^2} = z - a \arctan \frac{z}{a} = \sin t - a \arctan \frac{\sin t}{a}$$

$$= \sqrt{x^2 - a^2} - a \arctan \frac{\sqrt{x^2 - a^2}}{a} + C$$

1204\*

$$\int \frac{dx}{x\sqrt{x^2-1}} =$$

$$\left\{ \begin{array}{l} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{array} \right\}$$

$$= \int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}-1}} = -\int \frac{\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1-t^2}{t^2}}} = -\int \frac{\frac{dt}{t}}{\sqrt{\frac{1-t^2}{t^2}}} = -\int \frac{\frac{dt}{t}}{\sqrt{1-t^2}} = -\int \frac{dt}{\sqrt{1-t^2}} = -\arcsin t =$$

$$-\arcsin \frac{1}{x} + C$$

1205

$$\int \frac{\sqrt{x^2+1}}{x} dx$$

$$\left\{ \begin{array}{l} x = \sinh t \\ dx = \cosh t dt \end{array} \right\}$$

$$= \int \frac{\sqrt{\sinh^2 t + 1}}{\sinh t} \cosh t dt = \int \frac{\cosh^2 t}{\sinh t} dt = \int \frac{\sinh^2 t + 1}{\sinh t} dt = \int \sinh t + \int \frac{dt}{\sinh t}$$

$$= \cosh t - 2\operatorname{arctanh}(e^t) = \cosh \operatorname{arsinh} x - 2\operatorname{arctanh}(e^{\operatorname{arsinh} x}) + C$$

1206\*

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$\left\{ \begin{array}{l} x = 2\sin t \\ dx = 2\cos t dt \end{array} \right\}$$

$$= \int \frac{2\cos t dt}{(2\sin t)^2 \sqrt{4-(2\sin t)^2}} = \int \frac{2\cos t}{4\sin^2 t \sqrt{4-4\sin^2 t}} dt = \int \frac{2\cos t}{4\sin^2 t \sqrt{4-4\sin^2 t}} dt = \int \frac{2\cos t}{8\sin^2 t \sqrt{1-\sin^2 t}} dt = \int \frac{2\cos t}{8\sin^2 t \cos^2 t} dt$$

$$= \int \frac{2\cos t}{8\sin^2 t \cos t} dt = \frac{1}{4} \int \frac{dt}{\sin^2 t} = -\frac{1}{4} \tan t = -\frac{1}{4} \tan(\operatorname{arcsin} \frac{x}{2}) + C$$

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$$\left\{ \begin{array}{l} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{array} \right\}$$

$$\int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{4-\frac{1}{t^2}}{4-\frac{1}{t^2}}}} = -\int \frac{dt}{\sqrt{\frac{4t^2-1}{t^2}}} = -\int \frac{dt}{\frac{1}{t}\sqrt{4t^2-1}} = -\frac{1}{2} \int \frac{t}{\sqrt{t^2-\frac{1}{4}}} dt = -\frac{1}{2} \left( \frac{1}{2} \sqrt{4t^2-1} \right) =$$

$$-\frac{1}{4} \sqrt{4t^2-1}$$

$$= -\frac{1}{4} \sqrt{4(\frac{1}{x})^2 - 1} = -\frac{1}{4x} \sqrt{4-x^2} + C$$

1207

$$\int \sqrt{1-x^2} dx = \int \frac{1-x^2}{\sqrt{1-x^2}} dx$$

$$\int \frac{1-x^2}{\sqrt{1-x^2}} = (ax+b)\sqrt{1-x^2} + \int \frac{c}{\sqrt{1-x^2}} |'$$

$$\frac{1-x^2}{\sqrt{1-x^2}} = ((ax+b)\sqrt{1-x^2})' + \frac{c}{\sqrt{1-x^2}}$$

$$\frac{1-x^2}{\sqrt{1-x^2}} = a\sqrt{1-x^2} + \frac{-2x(ax+b)}{2\sqrt{1-x^2}} + \frac{c}{\sqrt{1-x^2}} |\sqrt{1-x^2}|$$

$$1-x^2 = a(1-x^2) - x(ax+b) + c$$

$$1-x^2 = a - 2ax^2 - xb + c$$

$$1-x^2 = x^2(-2a) + x(-b) + a + c$$

$$\{a = \frac{1}{2}, b = 0\}$$

$$a+c = 1 \dots c = \frac{1}{2}$$

$$I = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2} \int \frac{dx}{\sqrt{1-x^2}} = \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2} \arcsin x + C$$

1208

$$\int \frac{dx}{\sqrt{x(1-x)}}$$

$$\left\{ \begin{array}{l} x = \sin^2 t \\ dx = 2 \sin t \cos t dt \end{array} \right\}$$

$$= \int \frac{2 \sin t \cos t}{\sqrt{\sin^2 t(1-\sin^2 t)}} dt = 2 \int \frac{\sin t \cos t}{\sin t \sqrt{(1-\sin^2 t)}} dt = 2 \int \frac{\sin t \cos t}{\sin t \cos t} dt = 2 \int dt = 2t =$$

$$2 \arcsin \sqrt{x} + C$$

1209

$$\int \sqrt{a^2+x^2} dx$$

$$\left\{ \begin{array}{l} x = a \sinh t \\ dx = a \cosh t dt \end{array} \right\}$$

$$= \int \sqrt{a^2+a^2 \sinh^2 t} a \cosh t dt = a^2 \int \sqrt{1+\sinh^2 t} \cosh t dt$$

$$= a^2 \int \sqrt{\cosh^2 t} \cosh t dt = a^2 \int \cosh^2 t dt = a^2 \int \frac{\cosh 2t+1}{2} dt = \frac{a^2}{2} \int \cosh 2t dt + \frac{a^2}{2} \int dt$$

$$= \frac{1}{4} a^2 \sinh 2t + \frac{1}{2} a^2 t = \frac{1}{4} a^2 \sinh(2 \operatorname{arcsinh} \frac{x}{a}) + \frac{1}{2} a^2 \operatorname{arcsinh} \frac{x}{a}$$

$$= \frac{1}{2} x \sqrt{x^2+a^2} + \frac{1}{2} a^2 \operatorname{arcsinh} \frac{x}{a} + C$$

1210

$$\int \frac{x^2}{\sqrt{x^2-a^2}} dx$$

$$\left\{ \begin{array}{l} x = a \cosh t \\ dx = a \sinh t dt \end{array} \right\}$$

$$= \int \frac{a^2 \cosh^2 t}{\sqrt{a^2 \cosh^2 t - a^2}} dt = \int \frac{a^2 \cosh^2 t}{a \sqrt{\cosh^2 t - 1}} dt = a \int \frac{\cosh^2 t}{\sqrt{\cosh^2 t - 1}} dt$$

$$= a \int \frac{1+\sinh^2 t}{\sinh t} dt = a \int \frac{dt}{\sinh t} + a \int \sinh t dt = a \cosh t + a \ln \tanh \frac{t}{2}$$

$$= a \cosh(\operatorname{arccosh} \frac{x}{a}) + a \ln \tanh \frac{(\operatorname{arccosh} \frac{x}{a})}{2} = x + a \ln(\tanh(\frac{1}{2} \operatorname{arccosh} \frac{x}{a})) + C$$

1211

$$\int \ln x dx$$

$$\left\{ \begin{array}{l} dx = dv \quad \ln x = u \\ x = v \quad \frac{dx}{x} = du \end{array} \right\}$$

$$= x \ln x - \int x \frac{dx}{x} = x \ln x - x + C$$

1212

$$\int \operatorname{arctan} x dx$$

$$\left\{ \begin{array}{l} \operatorname{arctan} x = u \quad dx = dv \\ \frac{dx}{1+x^2} = du \quad x = v \end{array} \right\}$$

$$= x \operatorname{arctan} x - \int \frac{x}{1+x^2} dx$$

$$\left\{ \begin{array}{l} 1+x^2 = t \\ 2x dx = dt \end{array} \right\}$$

$$= x \operatorname{arctan} x - \int \frac{dt}{t} = x \operatorname{arctan} x - \frac{1}{2} \ln t = x \operatorname{arctan} x - \frac{1}{2} \ln(1+x^2) + C$$

1213

$$\int \operatorname{arcsin} x dx$$

$$\left\{ \begin{array}{l} \operatorname{arcsin} x = u \quad dx = dv \\ \frac{dx}{\sqrt{1-x^2}} = du \quad x = v \end{array} \right\}$$

$$= x \operatorname{arcsin} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$\left\{ \begin{array}{l} 1-x^2 = t \\ -2x dx = dt \end{array} \right\}$$

$$= x \operatorname{arcsin} x - \int \frac{dt}{\sqrt{t}} = x \operatorname{arcsin} x + \frac{1}{2} \int \frac{dt}{\sqrt{t}} = x \operatorname{arcsin} x + \sqrt{t} = x \operatorname{arcsin} x + \sqrt{1-x^2} + C$$

1214

$$\int x \sin x dx$$

$$\begin{cases} x = u & \sin x dx = dv \\ dx = du & -\cos x = v \end{cases}$$

$$= -x \cos x - \int -\cos x dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$$

1215

$$\int x \cos 3x dx$$

$$\begin{cases} 3x = t \\ 3dx = dt \end{cases}$$

$$= \frac{1}{9} \int t \cos t dt$$

$$\begin{cases} t = u & \cos t dt = dv \\ dt = du & \sin t = v \end{cases}$$

$$= \frac{1}{9} \left( t \sin t - \int \sin t dt \right)$$

$$= \frac{1}{9} t \sin t + \frac{1}{9} \cos t = \frac{1}{9} (3x) \sin(3x) + \frac{1}{9} \cos(3x)$$

$$= \frac{1}{3} x \sin 3x + \frac{1}{9} \cos 3x + C$$

1216

$$\int \frac{x}{e^x} dx$$

$$\begin{cases} x = u & e^{-x} dx = dv \\ dx = du & -e^{-x} = v \end{cases}$$

$$= -xe^{-x} - \int -e^{-x} dx = -xe^{-x} - e^{-x} + C$$

1217

$$\int x 2^{-x} dx$$

$$\begin{cases} x = u & 2^{-x} dx = dv \\ dx = du & -\frac{1}{\ln 2} 2^{-x} = v \end{cases}$$

$$= -\frac{1}{\ln 2} x 2^{-x} - \int -\frac{1}{\ln 2} 2^{-x} dx = -\frac{1}{\ln 2} x 2^{-x} + \frac{1}{\ln 2} \int 2^{-x} dx = -\frac{1}{\ln 2} x 2^{-x} - \frac{1}{\ln^2 2} 2^{-x} + C$$

1218 \*

$$\int x^2 e^{3x} dx$$

$$\begin{cases} 3x = t \\ 3dx = dt \end{cases}$$

$$= \int \frac{t^2}{9} e^t \frac{dt}{3} = \frac{1}{27} \int t^2 e^t dt$$

$$\begin{cases} e^t dt = dv & t^2 = u \\ e^t = v & 2tdt = du \end{cases}$$

$$I = \frac{1}{27} \left( t^2 e^t - 2 \int e^t t dt \right)$$

$$I_1 = \int e^t t dt$$

$$\begin{cases} t = u & e^t dt = dv \\ dt = du & e^t = v \end{cases}$$

$$= te^t - \int e^t dt = te^t - e^t$$

$$I = \frac{1}{27} (t^2 e^t - 2I_1) = \frac{1}{27} (t^2 e^t - 2(te^t - e^t)) = \frac{1}{27} t^2 e^t - \frac{2}{27} te^t + \frac{2}{27} e^t$$

$$= \frac{1}{27} (3x)^2 e^{(3x)} - \frac{2}{27} (3x) e^{(3x)} + \frac{2}{27} e^{(3x)} = \frac{1}{3} x^2 e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + C$$

1219 \*

$$\int (x^2 - 2x + 5) e^{-x} dx = \int x^2 e^{-x} dx - 2 \int x e^{-x} dx + 5 \int e^{-x} dx$$

$$\begin{cases} x^2 = u & e^{-x} dx = dv \\ 2x dx = du & -e^{-x} = v \end{cases}$$

$$I_1 = -e^{-x} x^2 - \int -e^{-x} 2x dx = -e^{-x} x^2 + 2 \int x e^{-x} dx$$

$$\begin{cases} x = u & e^{-x} dx = dv \\ dx = du & -e^{-x} = v \end{cases}$$

$$I_{12} = -e^{-x} x - \int -e^{-x} dx = -e^{-x} x + \int e^{-x} dx = -e^{-x} x - e^{-x}$$

$$I_1 = -e^{-x} x^2 + 2(-e^{-x} x - e^{-x}) = -e^{-x} x^2 - 2e^{-x} x - 2e^{-x}$$

$$I_2 = 2 \int x e^{-x} dx = 2(-e^{-x} x - e^{-x}) = -2e^{-x} x - 2e^{-x}$$

$$I_3 = -5e^{-x}$$

$$I = I_1 - I_2 + I_3 = -e^{-x} x^2 - 2e^{-x} x - 2e^{-x} - (-2e^{-x} x - 2e^{-x}) - 5e^{-x}$$

$$I = -e^{-x} x^2 - 5e^{-x} + C$$

1220\*

$$\int x^3 e^{-\frac{x}{3}} dx$$

$$\begin{cases} -\frac{x}{3} = t \\ -\frac{dx}{3} = dt \end{cases}$$

$$= \int -27t^3 e^t (-3dt) = 81 \int t^3 e^t dt = 81 I_1$$

$$I_1 = \int t^3 e^t dt$$

$$\begin{cases} e^t dt = dv & t^3 = u \\ e^t = v & 3t^2 dt = du \end{cases}$$

$$= t^3 e^t - 3 \int e^t t^2 dt = t^3 e^t - 3I_2$$

$$I_2 = \int e^t t^2 dt$$

$$\begin{cases} e^t dt = dv & t^2 = u \\ e^t = v & 2tdt = du \end{cases}$$

$$I_2 = t^2 e^t - 2 \int e^t t dt = t^2 e^t - 2I_3$$

$$I_3 = \int e^t t dt$$

$$\begin{cases} e^t dt = dv & t = u \\ e^t = v & dt = du \end{cases}$$

$$= te^t - \int e^t dt = te^t - e^t$$

$$\begin{aligned}
I_2 &= t^2 e^t - 2I_3 = t^2 e^t - 2(te^t - e^t) = t^2 e^t - 2te^t + 2e^t \\
I_1 &= t^3 e^t - 3I_2 = t^3 e^t - 3(t^2 e^t - 2te^t + 2e^t) = t^3 e^t - 3t^2 e^t + 6te^t - 6e^t \\
I &= 8I_1 = 81(t^3 e^t - 3t^2 e^t + 6te^t - 6e^t) \\
&= 81\left(-\frac{x}{3}\right)^3 e^{-\frac{x}{3}} - 243\left(-\frac{x}{3}\right)^2 e^{-\frac{x}{3}} + 486\left(-\frac{x}{3}\right)e^{-\frac{x}{3}} - 486e^{-\frac{x}{3}} \\
&= -3x^3 e^{-\frac{1}{3}x} - 27x^2 e^{-\frac{1}{3}x} - 162xe^{-\frac{1}{3}x} - 486e^{-\frac{1}{3}x} + C
\end{aligned}$$

1221

$$\begin{aligned}
\int x \sin x \cos x dx &= \frac{1}{2} \int x \sin 2x dx \\
\left\{ \begin{array}{l} 2x = t \\ 2dx = dt \end{array} \right\} \\
&= \frac{1}{2} \int \frac{t}{2} \sin t \frac{dt}{2} = \frac{1}{8} \int t \sin t dt \\
\left\{ \begin{array}{l} u = t \quad dv = \sin t dt \\ du = dt \quad v = -\cos t \end{array} \right\} \\
&= \frac{1}{8} \left( -t \cos t - \int -\cos t dt \right) = \frac{1}{8} \left( -t \cos t + \int \cos t dt \right) = -\frac{1}{8} t \cos t + \frac{1}{8} \sin t \\
&= -\frac{1}{8} (2x) \cos(2x) + \frac{1}{8} \sin(2x) = -\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C
\end{aligned}$$

1222\*

$$\begin{aligned}
\int (x^2 + 5x + 6) \cos 2x dx & \\
\left\{ \begin{array}{l} 2x = t \\ 2dx = dt \end{array} \right\} \\
&= \int \left( \frac{t^2}{4} + \frac{5}{2}t + 6 \right) \cos t \frac{dt}{2} = \frac{1}{8} \int t^2 \cos t dt + \frac{5}{4} \int t \cos t dt + 3 \int \cos t dt \\
&= 3 \sin t + \frac{1}{8} \int t^2 \cos t dt + \frac{5}{4} \int t \cos t dt \\
\left\{ \begin{array}{l} t^2 = u \quad \cos t dt = dv \\ 2tdt = du \quad \sin t = v \end{array} \right\} \left\{ \begin{array}{l} t = u \quad \cos t dt = dv \\ dt = du \quad \sin t = v \end{array} \right\} \\
I_a &= t^2 \sin t - 2 \int t \sin t dt \\
I_b &= t \sin t - \int \sin t dt = t \sin t + \cos t \\
I_{aa} &= \int t \sin t dt \\
&= -t \cos t - \int -\cos t dt = -t \cos t + \sin t
\end{aligned}$$

$$\begin{aligned}
I_a &= t^2 \sin t - 2(-t \cos t + \sin t) = t^2 \sin t + 2t \cos t - 2 \sin t \\
I &= 3 \sin t + \frac{1}{8} I_a + \frac{5}{4} I_b = 3 \sin t + \frac{1}{8} (t^2 \sin t + 2t \cos t - 2 \sin t) + \frac{5}{4} (t \sin t + \cos t) = \\
&\frac{11}{4} \sin t + \frac{1}{8} t^2 \sin t + \frac{1}{4} t \cos t + \frac{5}{4} t \sin t + \frac{5}{4} \cos t \\
&= \frac{11}{4} \sin(2x) + \frac{1}{8} (2x)^2 \sin(2x) + \frac{1}{4} (2x) \cos(2x) + \frac{5}{4} (2x) \sin(2x) + \frac{5}{4} \cos(2x) \\
&= \frac{11}{4} \sin 2x + \frac{1}{2} x^2 \sin 2x + \frac{1}{2} x \cos 2x + \frac{5}{2} x \sin 2x + \frac{5}{4} \cos 2x + C
\end{aligned}$$

1223

$$\int x^2 \ln x dx$$

$$\begin{aligned}
&\left\{ \begin{array}{l} \ln x = u \quad x^2 dx = dv \\ \frac{dx}{x} = du \quad \frac{x^3}{3} = v \end{array} \right\} \\
&= \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \frac{dx}{x} = \frac{x^3}{3} \ln x - \frac{1}{3} \int x^2 dx = \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C
\end{aligned}$$

1224

$$\begin{aligned}
\int \ln^2 x dx & \\
\left\{ \begin{array}{l} \ln^2 x = u \quad dx = dv \\ \frac{2 \ln x}{x} dx = du \quad x = v \end{array} \right\} \\
&= x \ln^2 x - \int x \frac{2 \ln x}{x} dx = x \ln^2 x - 2 \int \ln x dx \\
I_1 &= \int \ln x dx \\
\left\{ \begin{array}{l} \ln x = u \quad dx = dv \\ \frac{dx}{x} = du \quad x = v \end{array} \right\} \\
I_1 &= x \ln x - \int x \frac{dx}{x} = x \ln x - x \\
I &= x \ln^2 x - 2(x \ln x - x) = x \ln^2 x - 2x \ln x + 2x + C
\end{aligned}$$

1225

$$\begin{aligned}
\int \frac{\ln x}{x^3} dx & \\
\left\{ \begin{array}{l} \ln x = u \quad x^{-3} dx = dv \\ \frac{dx}{x} = du \quad \frac{x^{-2}}{-2} = v \end{array} \right\} \\
&= \ln x \frac{x^{-2}}{-2} - \int \frac{x^{-2}}{-2} \frac{dx}{x} = -\frac{1}{2} \frac{\ln x}{x^2} + \frac{1}{2} \int \frac{dx}{x^3} = -\frac{1}{2} \frac{\ln x}{x^2} - \frac{1}{4x^2} + C
\end{aligned}$$

1226

$$\begin{aligned}
\int \frac{\ln x}{\sqrt{x}} dx & \\
\left\{ \begin{array}{l} \ln x = u \quad \frac{dx}{\sqrt{x}} = dv \\ \frac{dx}{x} = du \quad 2\sqrt{x} = v \end{array} \right\} \\
&= 2\sqrt{x} \ln x - 2 \int \sqrt{x} \frac{dx}{x} = 2\sqrt{x} \ln x - 2 \int \frac{dx}{\sqrt{x}} = 2\sqrt{x} \ln x - 4\sqrt{x} + C
\end{aligned}$$

1227

$$\begin{aligned}
\int x \arctan x dx & \\
\left\{ \begin{array}{l} \arctan x = u \quad x dx = dv \\ \frac{dx}{1+x^2} = du \quad \frac{x^2}{2} = v \end{array} \right\} \\
&= \frac{x^2}{2} \arctan x - \int \frac{x^2}{2} \frac{dx}{1+x^2} = \frac{x^2}{2} \arctan x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx = \frac{x^2}{2} \arctan x - \frac{1}{2} \int \frac{x^2+1-1}{1+x^2} dx \\
&= \frac{x^2}{2} \arctan x - \frac{1}{2} \int dx + \frac{1}{2} \int \frac{dx}{1+x^2} = \frac{1}{2} x^2 \arctan x - \frac{1}{2} x + \frac{1}{2} \arctan x + C
\end{aligned}$$

1228

$$\int x \arcsin x dx$$

$$\begin{aligned}
& \left\{ \begin{array}{l} xdx = dv \quad \arcsinx = u \\ \frac{1}{2}x^2 = v \quad \frac{dx}{\sqrt{1-x^2}} = du \end{array} \right\} \\
& = \frac{1}{2}x^2 \arcsinx - \int \frac{1}{2}x^2 \frac{dx}{\sqrt{1-x^2}} \\
& = \frac{1}{2}x^2 \arcsinx + \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} dx = \frac{1}{2}x^2 \arcsinx + \frac{1}{2} \int \sqrt{1-x^2} dx - \frac{1}{2} \int \frac{dx}{\sqrt{1-x^2}} \\
& = \frac{1}{2}x^2 \arcsinx + \frac{1}{4}x\sqrt{(1-x^2)} - \frac{1}{4} \arcsinx + C
\end{aligned}$$

1229

$$\begin{aligned}
& \int \ln(x + \sqrt{1+x^2}) dx \\
& \left\{ \begin{array}{l} dx = dv \quad \ln(x + \sqrt{1+x^2}) = u \\ x = v \quad \frac{dx}{\sqrt{1+x^2}} = du \end{array} \right\} \\
& = x \ln(x + \sqrt{1+x^2}) - \int x \frac{dx}{\sqrt{1+x^2}} = x \ln(x + \sqrt{1+x^2}) - \int \frac{x}{\sqrt{1+x^2}} dx \\
& = x \ln(x + \sqrt{1+x^2}) - \int \frac{x}{\sqrt{1+x^2}} dx \\
& \left\{ \begin{array}{l} 1+x^2 = t \\ 2xdx = dt \end{array} \right\} \\
& = x \ln(x + \sqrt{1+x^2}) - \int \frac{\frac{dt}{2}}{\sqrt{t}} \\
& = x \ln(x + \sqrt{1+x^2}) - \sqrt{t} = x \ln(x + \sqrt{(1+x^2)}) - \sqrt{1+x^2} + C
\end{aligned}$$

1230

$$\begin{aligned}
& \int \frac{xdx}{\sin^2 x} \\
& \left\{ \begin{array}{l} x = u \quad \frac{dx}{\sin^2 x} = dv \\ dx = du \quad -\cot x = v \end{array} \right\} \\
& = -x \cot x - \int -\cot x dx \\
& = -x \cot x + \int \cot x dx = -x \cot x + \ln(\sin x) + C
\end{aligned}$$

1231

$$\begin{aligned}
I &= \int \frac{x \cos x}{\sin^2 x} \\
& \left\{ \begin{array}{l} \frac{\cos x}{\sin^2 x} dx = dv \quad x = u \\ -\frac{1}{\sin x} = v \quad dx = du \end{array} \right\} \\
I_1 &= \int \frac{\cos x}{\sin^2 x} \\
& \left\{ \begin{array}{l} \sin x = t \\ \cos x dx = dt \end{array} \right\} \\
& = \int \frac{dt}{t^2} = -\frac{1}{t} = -\frac{1}{\sin x} \\
I &= -x \frac{1}{\sin x} - \int -\frac{1}{\sin x} dx = -\frac{x}{\sin x} + \int \frac{dx}{\sin x} = -\frac{x}{\sin x} + \ln \tan \frac{x}{2} + C
\end{aligned}$$

1232

$$\begin{aligned}
& \int e^x \sin x dx \\
& \left\{ \begin{array}{l} e^x dx = dv \quad \sin x = u \\ e^x = v \quad \cos x dx = du \end{array} \right\} \\
I &= e^x \sin x - \int e^x \cos x dx \\
I_1 &= \int e^x \cos x dx \\
& \left\{ \begin{array}{l} e^x dx = dv \quad \cos x = u \\ e^x = v \quad -\sin x dx = du \end{array} \right\} \\
& = e^x \cos x - \int e^x - \sin x dx \\
I_1 &= e^x \cos x + \int e^x \sin x dx \\
I &= e^x \sin x - (e^x \cos x + \int e^x \sin x dx) = e^x \sin x - e^x \cos x - I \\
2I &= e^x \sin x - e^x \cos x \\
I &= \frac{e^x \sin x - e^x \cos x}{2} + C \\
& \int e^x \sin x dx \\
& \left\{ \begin{array}{l} e^x = u \quad \sin x dx = dv \\ e^x dx = du \quad -\cos x = v \end{array} \right\} \\
& = -e^x \cos x - \int -\cos x e^x dx \\
& = -e^x \cos x + \int \cos x e^x dx \\
& \left\{ \begin{array}{l} e^x = u \quad \cos x dx = dv \\ e^x dx = du \quad \sin x = v \end{array} \right\} \\
I &= -e^x \cos x + (e^x \sin x - \int \sin x e^x dx) \\
I &= -e^x \cos x + e^x \sin x - I \\
2I &= -e^x \cos x + e^x \sin x \\
I &= \frac{-e^x \cos x + e^x \sin x}{2} + C
\end{aligned}$$

1233

$$\begin{aligned}
& \int 3^x \cos x dx \\
& \left\{ \begin{array}{l} 3^x dx = dv \quad \cos x = u \\ \frac{3^x}{\ln 3} = v \quad -\sin x = du \end{array} \right\} \\
& = \frac{3^x}{\ln 3} \cos x - \int -\sin x \frac{3^x}{\ln 3} dx \\
I &= \frac{3^x}{\ln 3} \cos x + \frac{1}{\ln 3} \int 3^x \sin x dx \\
& \left\{ \begin{array}{l} 3^x dx = dv \quad \sin x = u \\ \frac{3^x}{\ln 3} = v \quad \cos x = du \end{array} \right\} \\
I &= \frac{3^x}{\ln 3} \cos x + \frac{1}{\ln 3} \left( \sin x \frac{3^x}{\ln 3} - \int \frac{3^x}{\ln 3} \cos x \right) = \frac{3^x}{\ln 3} \cos x + \frac{3^x}{\ln^2 3} \sin x - \frac{1}{\ln^2 3} \int 3^x \cos x \\
I &= \frac{3^x}{\ln 3} \cos x + \frac{3^x}{\ln^2 3} \sin x - \frac{1}{\ln^2 3} I \\
I \left( 1 + \frac{1}{\ln^2 3} \right) &= \frac{3^x}{\ln 3} \cos x + \frac{3^x}{\ln^2 3} \sin x
\end{aligned}$$

$$I = \frac{\frac{3^x}{\ln 3} \cos x + \frac{3^x}{\ln^2 3} \sin x}{\frac{\ln^2 3 + 1}{\ln^2 3}} = \frac{\frac{3^x}{\ln 3} \cos x + \frac{3^x}{\ln^2 3} \sin x}{\ln^2 3 + 1} \ln^2 3 = 3^x \frac{\cos x \ln 3 + \sin x}{\ln^2 3 + 1} + C$$

1234

$$\begin{aligned} & \int e^{ax} \sin bxdx \\ & \left\{ \begin{array}{l} bx = t \\ bdx = dt \end{array} \right\} \\ & = \int e^{\frac{at}{b}} \sin t \frac{dt}{b} = \frac{1}{b} \int e^{\frac{at}{b}} \sin t dt \\ & \left\{ \begin{array}{l} e^{\frac{at}{b}} = u \quad \sin t dt = dv \\ \frac{a}{b} e^{\frac{at}{b}} dt = du \quad -\cos t = v \end{array} \right\} \\ & = \frac{1}{b} \left( -e^{\frac{at}{b}} \cos t + \frac{a}{b} \int e^{\frac{at}{b}} \cos t dt \right) \\ & \left\{ \begin{array}{l} e^{\frac{at}{b}} = u \quad \cos t dt = dv \\ \frac{a}{b} e^{\frac{at}{b}} dt = du \quad \sin t = v \end{array} \right\} \\ I & = \left( -\frac{1}{b} e^{\frac{at}{b}} \cos t + \frac{a}{b^2} \left( e^{\frac{at}{b}} \sin t - a \frac{1}{b} \int e^{\frac{at}{b}} \sin t dt \right) \right) \\ I & = \left( -\frac{1}{b} e^{\frac{at}{b}} \cos t + \frac{a}{b^2} \left( e^{\frac{at}{b}} \sin t - at \right) \right) \\ I & = \left( -\frac{1}{b} e^{\frac{at}{b}} \cos t + \frac{a}{b^2} e^{\frac{at}{b}} \sin t - \frac{a^2}{b^2} I \right) \\ I \left( 1 + \frac{a^2}{b^2} \right) & = -\frac{1}{b} e^{\frac{at}{b}} \cos t + \frac{a}{b^2} e^{\frac{at}{b}} \sin t \\ I & = \frac{-\frac{1}{b} e^{\frac{at}{b}} \cos t + \frac{a}{b^2} e^{\frac{at}{b}} \sin t}{\frac{a^2+b^2}{b^2}} = e^{\frac{at}{b}} \frac{-b \cos t + a \sin t}{a^2+b^2} = e^{\frac{(bx)}{b}} \frac{-b \cos(bx) + a \sin(bx)}{a^2+b^2} \\ I & = \frac{-b \cos(bx) + a \sin(bx)}{a^2+b^2} e^{ax} + C \end{aligned}$$

1235

$$\begin{aligned} & \int \sin(\ln x) dx \\ & \left\{ \begin{array}{l} \ln x = t \\ \frac{dx}{x} = dt \end{array} \right\} \\ & = \int x \sin t dt = \int e^t \sin t dt \\ & \left\{ \begin{array}{l} e^t dt = dv \quad \sin t = u \\ e^t = v \quad \cos t dt = du \end{array} \right\} \\ I & = e^t \sin t - \int e^t \cos t dt \\ I_1 & = \int e^t \cos t dt \\ & \left\{ \begin{array}{l} e^t dt = dv \quad \cos t = u \\ e^t = v \quad -\sin t dt = du \end{array} \right\} \\ I_1 & = e^t \cos t - \int e^t \sin t dt = e^t \cos t + \int e^t \sin t dt \\ I & = e^t \sin t - (e^t \cos t + \int e^t \sin t dt) = e^t \sin t - e^t \cos t - I \\ 2I & = e^t \sin t - e^t \cos t \\ I & = \frac{e^t \sin t - e^t \cos t}{2} = \frac{e^{\ln x} \sin \ln x - e^{\ln x} \cos \ln x}{2} = \frac{1}{2} x \sin(\ln x) - \frac{1}{2} x \cos(\ln x) + C \end{aligned}$$

1236

$$\begin{aligned} & \int x^3 e^{-x^2} dx \\ & \left\{ \begin{array}{l} -x^2 = t \\ -2x dx = dt \end{array} \right\} \\ & = \int x^3 e^t \frac{dt}{-2x} = \frac{1}{2} \int -x^2 e^t dt = \frac{1}{2} \int te^t dt \\ & \left\{ \begin{array}{l} t = u \quad e^t dt = dv \\ dt = du \quad e^t = v \end{array} \right\} \\ & = \frac{1}{2} (te^t - \int e^t dt) = \frac{1}{2} te^t - \frac{1}{2} e^t \\ & = \frac{1}{2} (-x^2) e^{-x^2} - \frac{1}{2} e^{-x^2} = -\frac{1}{2} x^2 e^{-x^2} - \frac{1}{2} e^{-x^2} + C \end{aligned}$$

1237

$$\begin{aligned} & \int e^{\sqrt{x}} dx \\ & \left\{ \begin{array}{l} \sqrt{x} = t^2 \\ x = t^2 \\ dx = 2tdt \end{array} \right\} \\ & = 2 \int e^t t dt \\ & \left\{ \begin{array}{l} t = u \quad e^t dt = dv \\ dt = du \quad e^t = v \end{array} \right\} \\ & = 2(te^t - \int e^t dt) = 2te^t - 2e^t = 2\sqrt{x} e^{\sqrt{x}} - 2e^{\sqrt{x}} + C \end{aligned}$$

1238

$$\begin{aligned} & \int (x^2 - 2x + 3) \ln x dx \\ & \left\{ \begin{array}{l} (x^2 - 2x + 3) dx = dv \quad \ln x = u \\ \frac{x^3}{3} - x^2 + 3x = v \quad \frac{dx}{x} = du \end{array} \right\} \\ & = \left( \frac{x^3}{3} - x^2 + 3x \right) \ln x - \int \left( \frac{x^3}{3} - x^2 + 3x \right) \left( \frac{dx}{x} \right) = \left( \frac{x^3}{3} - x^2 + 3x \right) \ln x - \int \left( \frac{x^2}{3} - x + 3 \right) dx \\ & = \left( \frac{x^3}{3} - x^2 + 3x \right) \ln x - \frac{x^3}{9} + \frac{x^2}{2} - 3x + C \end{aligned}$$

1239

$$\begin{aligned} & \int x \ln \frac{1-x}{1+x} dx \\ & \left\{ \begin{array}{l} \ln \frac{1-x}{1+x} = u \quad dv = x dx \\ \frac{2}{x^2-1} dx = du \quad v = \frac{x^2}{2} \end{array} \right\} \\ & = \frac{x^2}{2} \ln \frac{1-x}{1+x} - \int \frac{x^2}{2} \frac{2}{x^2-1} dx = \frac{x^2}{2} \ln \frac{1-x}{1+x} - \int \frac{x^2}{x^2-1} dx = \frac{x^2}{2} \ln \frac{1-x}{1+x} - \int \frac{x^2-1+1}{x^2-1} dx \\ & = \frac{x^2}{2} \ln \frac{1-x}{1+x} - \int dx - \int \frac{dx}{x^2-1} = \frac{1}{2} x^2 \ln \frac{1-x}{1+x} - x + \arctan x + C \end{aligned}$$

1240

$$\int \frac{\ln^2 x}{x^2} dx$$

$$\begin{aligned} & \left\{ \begin{array}{l} \ln^2 x = u \\ \frac{2 \ln x}{x} dx = du \end{array} \right. \quad dv = \frac{dx}{x^2} \\ & \left. \begin{array}{l} v = -\frac{1}{x} \\ \frac{2 \ln x}{x} dx = du \end{array} \right\} \\ & = -\frac{1}{x} \ln^2 x - \int \left( -\frac{1}{x} \right) \frac{2 \ln x}{x} dx \\ & = -\frac{1}{x} \ln^2 x + 2 \int \frac{\ln x}{x^2} dx \\ & \left\{ \begin{array}{l} \ln x = u \\ \frac{dx}{x} = du \end{array} \right. \quad dv = \frac{dx}{x^2} \\ & \left. \begin{array}{l} v = -\frac{1}{x} \\ \frac{dx}{x} = du \end{array} \right\} \\ & = -\frac{1}{x} \ln^2 x + 2 \left( -\frac{1}{x} \ln x - \int \left( -\frac{1}{x} \right) \frac{dx}{x} \right) \\ & = -\frac{1}{x} \ln^2 x + 2 \left( -\frac{1}{x} \ln x + \int \frac{dx}{x^2} \right) = -\frac{1}{x} \ln^2 x - \frac{2}{x} \ln x - \frac{2}{x} + C \end{aligned}$$

1241

$$\begin{aligned} & \int \frac{\ln(\ln x)}{x} dx \\ & \left\{ \begin{array}{l} \ln(\ln x) = u \\ \frac{dx}{x} = dv \end{array} \right. \\ & \left. \begin{array}{l} \frac{dx}{x \ln x} = du \\ \ln x = v \end{array} \right\} \\ & = \ln x \ln(\ln x) - \int \ln x \frac{dx}{x \ln x} = \ln x \ln(\ln x) - \int \frac{dx}{x} = \ln x \ln(\ln x) - \ln x + C \end{aligned}$$

1242

$$\begin{aligned} & \int x^2 \arctan 3x dx \\ & \left\{ \begin{array}{l} 3x = t \\ 3dx = dt \end{array} \right. \\ & = \int \frac{t^2}{9} \arctan t \frac{dt}{3} = \frac{1}{27} \int t^2 \arctan t dt \\ & \left\{ \begin{array}{l} \arctan t = u \\ \frac{dt}{1+t^2} = du \end{array} \right. \quad t^2 dt = dv \\ & \left. \begin{array}{l} t^3 = v \\ \frac{dt}{1+t^2} = du \end{array} \right\} \\ & = \frac{1}{27} \left( \frac{t^3}{3} \arctan t - \int \frac{t^3}{3} \frac{dt}{1+t^2} \right) \\ & = \frac{1}{27} \left( \frac{t^3}{3} \arctan t - \frac{1}{3} \int \frac{t^3}{1+t^2} dt \right) = \frac{1}{81} \left( t^3 \arctan t - \int \frac{t^3}{1+t^2} dt \right) \\ & \left\{ \begin{array}{l} 1+t^2 = z \\ 2tdt = dz \end{array} \right. \\ & = \frac{1}{81} \left( t^3 \arctan t - \int \frac{t^3}{z} \frac{dz}{2t} \right) \\ & = \frac{1}{81} \left( t^3 \arctan t - \frac{1}{2} \int \frac{t^2}{z} dz \right) = \frac{1}{81} \left( t^3 \arctan t - \frac{1}{2} \int \frac{z-1}{z} dz \right) \\ & = \frac{1}{81} \left( t^3 \arctan t - \frac{1}{2} \int dz + \frac{1}{2} \int \frac{dz}{z} \right) = \frac{1}{81} \left( t^3 \arctan t - \frac{1}{2} z + \frac{1}{2} \ln z \right) \\ & = \frac{1}{81} \left( t^3 \arctan t - \frac{1}{2} (1+t^2) + \frac{1}{2} \ln(1+t^2) \right) \\ & = \frac{1}{81} ((3x)^3 \arctan(3x) - \frac{1}{2} (1+(3x)^2) + \frac{1}{2} \ln(1+(3x)^2)) \\ & = \frac{1}{81} (27x^3 \arctan 3x - \frac{1}{2} - \frac{9}{2}x^2 + \frac{1}{2} \ln(1+9x^2)) \\ & = \frac{1}{3}x^3 \arctan 3x - \frac{1}{162} - \frac{1}{18}x^2 + \frac{1}{162} \ln(1+9x^2) + C \end{aligned}$$

1243

$$\int x \arctan^2 x dx$$

$$\begin{aligned} & \left\{ \begin{array}{l} \arctan^2 x = u \\ \frac{2 \arctan x}{1+x^2} dx = du \end{array} \right. \quad x dx = dv \\ & \left. \begin{array}{l} \frac{x^2}{1+x^2} = v \\ \frac{2 \arctan x}{1+x^2} dx = du \end{array} \right\} \\ & = \frac{x^2}{2} \arctan^2 x - \int \frac{x^2}{2} \frac{2 \arctan x}{1+x^2} dx = \frac{x^2}{2} \arctan^2 x - \int \frac{x^2 \arctan x}{1+x^2} dx \\ & \left\{ \begin{array}{l} \arctan x = u \\ \frac{dx}{1+x^2} = du \end{array} \right. \quad \frac{x^2}{1+x^2} dx = dv \\ & \left. \begin{array}{l} x = \arctan x \\ \int \frac{x^2+1-1}{1+x^2} dx = x - \arctan x = v \end{array} \right\} \\ & = \frac{x^2}{2} \arctan^2 x - \left( (x - \arctan x) \arctan x - \int (x - \arctan x) \frac{dx}{1+x^2} \right) \\ & = \frac{x^2}{2} \arctan^2 x - \left( x \arctan x - \arctan^2 x - \int \frac{x}{1+x^2} dx + \int \frac{\arctan x}{1+x^2} dx \right) \\ & = \frac{x^2}{2} \arctan^2 x - x \arctan x + \arctan^2 x + I_1 - I_2 \\ & I_1 = \int \frac{x}{1+x^2} dx \\ & \left\{ \begin{array}{l} 1+x^2 = t \\ 2xdx = dt \end{array} \right. \\ & = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln t = \frac{1}{2} \ln(1+x^2) \\ & I_2 = \int \frac{\arctan x}{1+x^2} dx \\ & \left\{ \begin{array}{l} \arctan x = t \\ \frac{dx}{1+x^2} = dt \end{array} \right. \\ & = \int t dt = \frac{1}{2} t^2 = \frac{1}{2} \arctan^2 x \\ & = \frac{x^2}{2} \arctan^2 x - x \arctan x + \arctan^2 x + \frac{1}{2} \ln(1+x^2) - \frac{1}{2} \arctan^2 x \\ & = \frac{1}{2} x^2 \arctan^2 x - x \arctan x + \frac{1}{2} \arctan^2 x + \frac{1}{2} \ln(1+x^2) + C \end{aligned}$$

1244

$$\begin{aligned} & I = \int (\arcsin x)^2 dx = \\ & \left\{ \begin{array}{l} dv = dx \quad \arcsin^2 x = u \\ v = x \quad 2 \frac{\arcsin x}{\sqrt{1-x^2}} dx = du \end{array} \right\} \\ & I = x \arcsin^2 x - \int x 2 \frac{\arcsin x}{\sqrt{1-x^2}} dx \\ & I_v = \int x \frac{\arcsin x}{\sqrt{1-x^2}} \\ & \left\{ \begin{array}{l} \arcsin x = u \quad \frac{x}{\sqrt{1-x^2}} dx = dv \\ \frac{1}{\sqrt{1-x^2}} dx = du \quad -\sqrt{1-x^2} = v \end{array} \right. \\ & I_v = uv - \int v du = -\sqrt{1-x^2} \arcsin x - \int -\sqrt{1-x^2} \frac{1}{\sqrt{1-x^2}} dx = -\sqrt{1-x^2} \arcsin x + x \\ & I_v = \int \frac{x}{\sqrt{1-x^2}} \\ & \left\{ \begin{array}{l} 1-x^2 = t \\ -2x dx = dt \end{array} \right. \end{aligned}$$





$$I = \frac{9}{2} \arcsin \frac{x}{3} - \frac{x}{2} \sqrt{9-x^2} + C$$

1255

$$\int \frac{dx}{x^2+2x+5} = \int \frac{dx}{x^2+2x+1+4} = \int \frac{dx}{(x+1)^2+4} = \frac{1}{2} \arctan \frac{x+1}{2} + C$$

1256

$$\int \frac{dx}{x^2+2x} = \int \frac{dx}{x^2+2x+1-1} = \int \frac{dx}{(x+1)^2-1} = -\operatorname{arctanh}(x+1) + C$$

1257

$$\int \frac{dx}{3x^2-x+1} = \frac{1}{3} \int \frac{dx}{x^2-\frac{1}{3}x+\frac{1}{3}} = \frac{1}{3} \int \frac{dx}{\left(x-\frac{1}{6}\right)^2-\frac{1}{36}+\frac{1}{3}} = \frac{1}{3} \int \frac{dx}{\left(x-\frac{1}{6}\right)^2+\frac{11}{36}} = \frac{2}{\sqrt{11}} \arctan \frac{(6x-1)}{\sqrt{11}} + C$$

1258

$$\begin{aligned} \int \frac{xdx}{x^2-7x+13} &= \int \frac{xdx}{\left(x-\frac{7}{2}\right)^2-\frac{49}{4}+13} = \int \frac{x-\frac{7}{2}}{\left(x-\frac{7}{2}\right)^2+\frac{3}{4}} dx + \frac{7}{2} \int \frac{dx}{\left(x-\frac{7}{2}\right)^2+\frac{3}{4}} \\ &\quad \left\{ \begin{array}{l} x-\frac{7}{2}=t \\ dx=dt \end{array} \right\} \\ &= \int \frac{t}{t^2+\frac{3}{4}} dt + \frac{7}{2} \int \frac{dx}{\left(x-\frac{7}{2}\right)^2+\frac{3}{4}} = \frac{1}{2} \ln(4t^2+3) + \frac{7}{3} \sqrt{3} \arctan \frac{1}{3}(2x-7) \sqrt{3} \\ &= \frac{1}{2} \ln(4(x-\frac{7}{2})^2+3) + \frac{7}{3} \arctan \frac{2x-7}{\sqrt{3}} = \\ &\ln 2 + \frac{1}{2} \ln(x^2-7x+13) + \frac{7}{\sqrt{3}} \arctan \frac{2x-7}{\sqrt{3}} + C \end{aligned}$$

1259

$$\begin{aligned} \int \frac{3x-2}{x^2-4x+3} dx &= \int \frac{3x-2}{(x-2)^2-4+3} dx = \int \frac{3x-2}{(x-2)^2-1} dx = 3 \int \frac{xdx}{(x-2)^2-1} - 2 \int \frac{dx}{(x-2)^2-1} \\ &= 3 \int \frac{x-2+2}{(x-2)^2-1} dx - 2 \int \frac{dx}{(x-2)^2-1} = 3 \int \frac{x-2}{(x-2)^2-1} dx + 6 \int \frac{dx}{(x-2)^2-1} - 2 \int \frac{dx}{(x-2)^2-1} \\ &= 3 \int \frac{x-2}{(x-2)^2-1} dx + 4 \int \frac{dx}{(x-2)^2-1} \\ &\quad \left\{ \begin{array}{l} x-2=t \\ dx=dt \end{array} \right\} \quad \left\{ \begin{array}{l} t^2-1=z \\ 2tdt=dz \end{array} \right\} \\ &= 3 \int \frac{t}{t^2-1} dt + 4 \int \frac{dt}{t^2-1} = 3 \int \frac{t}{t^2-1} dt - 4 \operatorname{arctanh} t = \frac{3}{2} \int \frac{dt}{z} - 4 \operatorname{arctanh} t \\ &= \frac{3}{2} \ln z - 4 \operatorname{arctanh} t = \frac{3}{2} \ln(t^2-1) - 4 \operatorname{arctanh} t = \\ &\frac{3}{2} \ln((x-2)^2-1) - 4 \operatorname{arctanh}(x-2) \\ &= \frac{3}{2} \ln(x-1) + \frac{3}{2} \ln(x-3) - 4 \operatorname{arctanh}(x-2) \end{aligned}$$

1260

$$\begin{aligned} \int \frac{(x-1)^2}{x^2+3x+4} dx &= \int \frac{x^2-2x+1}{x^2+3x+4} dx = \int \frac{x^2+3x+4-2x+1-3x-4}{x^2+3x+4} dx = \int \frac{x^2+3x+4}{x^2+3x+4} dx - \int \frac{5x+3}{x^2+3x+4} dx \\ &= x - \frac{5}{2} \int \frac{2x+\frac{6}{5}}{x^2+3x+4} dx = x - \frac{5}{2} \int \frac{2x+\frac{6}{5}+3-\frac{3}{5}}{\left(x+\frac{3}{2}\right)^2-\frac{9}{4}+4} dx = x - \frac{5}{2} \int \frac{2x+3-\frac{3}{5}}{\left(x+\frac{3}{2}\right)^2+\frac{7}{4}} dx \\ &= x - \frac{5}{2} \int \frac{2x+3}{x^2+3x+4} dx + \frac{9}{2} \int \frac{dx}{\left(x+\frac{3}{2}\right)^2+\frac{7}{4}} \\ &\quad \left\{ \begin{array}{l} x^2+3x+4=t \\ (2x+3)dx=dt \end{array} \right. \quad \left\{ \begin{array}{l} x+\frac{3}{2}=z \\ dx=dz \end{array} \right. \\ &= x - \frac{5}{2} \int \frac{dt}{t} + \frac{9}{2} \int \frac{dz}{z^2+\frac{7}{4}} = x - \frac{5}{2} \ln t + \frac{9}{7} \sqrt{7} \arctan \frac{2}{7} z \sqrt{7} \end{aligned}$$

$$= x - \frac{5}{2} \ln(x^2+3x+4) + \frac{9}{\sqrt{7}} \arctan \frac{2x+3}{\sqrt{7}} + C$$

1261

$$\begin{aligned} \int \frac{x^2}{x^2-6x+10} dx &= \int \frac{x^2-6x+10+6x-10}{x^2-6x+10} dx = \int dx + \int \frac{6x-10}{(x-3)^2-9+10} dx = x + 3 \int \frac{2x-6-\frac{10}{3}+6}{(x-3)^2+1} dx \\ &= x + 3 \int \frac{2x-6}{x^2-6x+10} dx + 8 \int \frac{dx}{(x-3)^2+1} \\ &\quad \left\{ \begin{array}{l} x^2-6x+10=t \\ x-3=z \end{array} \right. \\ &\quad \left\{ \begin{array}{l} (2x-6)dx=dt \\ dx=dz \end{array} \right. \\ &= x + 3 \int \frac{dt}{t} + 8 \int \frac{dz}{z^2+1} = x + 3 \ln t + 8 \operatorname{arctan} z \\ &= x + 3 \ln(x^2-6x+10) + 8 \operatorname{arctan}(x-3) + C \end{aligned}$$

1262

$$\begin{aligned} \int \frac{dx}{\sqrt{3+3x-2x^2}} &= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{3}{2}+\frac{3}{2}x-x^2}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{\frac{33}{16}-(x-\frac{3}{4})^2}} \\ &\quad \left\{ \begin{array}{l} x-\frac{3}{4}=t \\ dx=dt \end{array} \right. \\ &= \frac{1}{\sqrt{2}} \int \frac{dt}{\sqrt{\frac{33}{16}-t^2}} = \frac{1}{\sqrt{2}} \arcsin \frac{4t}{\sqrt{33}} = \frac{1}{\sqrt{2}} \arcsin \frac{4x-3}{\sqrt{33}} + C \end{aligned}$$

1263

$$\begin{aligned} \int \frac{dx}{\sqrt{x-x^2}} &= \int \frac{dx}{\sqrt{\frac{1}{4}-(x-\frac{1}{2})^2}} \\ &\quad \left\{ \begin{array}{l} x-\frac{1}{2}=t \\ dx=dt \end{array} \right. \\ &= \int \frac{dt}{\sqrt{\frac{1}{4}-t^2}} = \arcsin 2t = \arcsin(2x-1) + C \end{aligned}$$

1264

$$\begin{aligned} \int \frac{dx}{\sqrt{x^2+px+q}} &= \int \frac{dx}{\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}+q}} = \int \frac{dx}{\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2}{4}+q}} = \int \frac{dx}{\sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2-4q}{4}}} \\ &\quad \left\{ \begin{array}{l} \left(x+\frac{p}{2}\right)=t \\ dx=dt \\ \frac{p^2-4q}{4}=z \end{array} \right. \\ &= \int \frac{dt}{\sqrt{t^2-z}} = \ln \left( t + \sqrt{t^2-z} \right) = \ln \left( x + \frac{p}{2} + \sqrt{\left(x+\frac{p}{2}\right)^2-\frac{p^2-4q}{4}} \right) = \\ &\ln \left( \frac{1}{2}p+x+\sqrt{x^2+xp+q} \right) + C \end{aligned}$$

1265

$$\begin{aligned} \int \frac{3x-6}{\sqrt{x^2-4x+5}} dx &= \frac{3}{2} \int \frac{2x-4}{\sqrt{x^2-4x+5}} dx \\ &\quad \left\{ \begin{array}{l} x^2-4x+5=t \\ (2x-4)dx=dt \end{array} \right. \end{aligned}$$

$$= \frac{3}{2} \int \frac{dt}{\sqrt{t}} = 3\sqrt{t} = 3\sqrt{x^2 - 4x + 5} + C$$

1266

$$\begin{aligned} \int \frac{2x-8}{\sqrt{1-x-x^2}} dx &= \int \frac{2x-8}{\sqrt{1-(x+\frac{1}{2})^2+\frac{1}{4}}} dx = 2 \int \frac{x-4}{\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}} dx = 2 \int \frac{x+\frac{1}{2}-4-\frac{1}{2}}{\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}} dx \\ &= 2 \int \frac{x+\frac{1}{2}}{\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}} dx - 9 \int \frac{dx}{\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}} \\ &\left\{ \begin{array}{l} \frac{5}{4} - (x+\frac{1}{2})^2 = t \quad x+\frac{1}{2} = z \\ -2(x+\frac{1}{2})dx = dt \quad dx = dz \end{array} \right\} \\ &= - \int \frac{dt}{\sqrt{t}} - 9 \int \frac{dz}{\sqrt{\frac{5}{4}-z^2}} = -2\sqrt{t} - 9 \arcsin \frac{2}{\sqrt{5}} z = -2\sqrt{1-x-x^2} - 9 \arcsin \frac{2x+1}{\sqrt{5}} + C \end{aligned}$$

1267

$$\begin{aligned} \int \frac{x}{\sqrt{5x^2-2x+1}} dx &= \frac{1}{10} \int \frac{10x-2+2}{\sqrt{x^2-\frac{2}{5}x+\frac{1}{5}}} dx = \frac{1}{10} \int \frac{10x-2}{\sqrt{5x^2-2x+1}} dx + \frac{1}{5\sqrt{5}} \int \frac{dx}{\sqrt{(x-\frac{1}{5})^2-\frac{1}{25}+\frac{1}{5}}} \\ &\left\{ \begin{array}{l} 5x^2 - 2x + 1 = t \quad x - \frac{1}{5} = z \\ (10x - 2)dx = dt \quad dx = dz \end{array} \right\} \\ &= \frac{1}{10} \int \frac{dt}{\sqrt{t}} + \frac{1}{5\sqrt{5}} \int \frac{dx}{\sqrt{(x-\frac{1}{5})^2+\frac{4}{25}}} \\ &= \frac{1}{10} 2t^{\frac{1}{2}} + \frac{1}{5\sqrt{5}} \int \frac{dz}{\sqrt{z^2+\frac{4}{25}}} = \frac{\sqrt{t}}{5} + \frac{\sqrt{5}}{25} \operatorname{arcsinh} \frac{5}{2} z = \frac{\sqrt{5x^2-2x+1}}{5} + \frac{\sqrt{5}}{25} \operatorname{arcsinh} \frac{5x-1}{2} + C \end{aligned}$$

1268

$$\begin{aligned} \int \frac{dx}{x\sqrt{1-x^2}} & \left\{ \begin{array}{l} \sqrt{1-x^2} = t \\ 1-x^2 = t^2 \\ -x dx = t dt \end{array} \right\} \\ \int -\frac{dt}{xt} &= - \int \frac{dt}{x^2 t} = - \int \frac{dt}{1-t^2} = - \operatorname{arctanh} t = - \operatorname{arctanh} \sqrt{1-x^2} + C \end{aligned}$$

1269

$$\begin{aligned} \int \frac{dx}{x\sqrt{x^2+x-1}} & \left\{ \begin{array}{l} x = \frac{1}{t} \\ dx = -\frac{dt}{t^2} \end{array} \right\} \\ \int \frac{-\frac{dt}{t^2}}{\frac{1}{t}\sqrt{\frac{1}{t^2}+\frac{1}{t}-1}} &= \int \frac{-\frac{dt}{t}}{\sqrt{\frac{1}{t^2}+\frac{1}{t}-1}} = \int \frac{-\frac{dt}{t}}{\sqrt{\frac{1+t-t^2}{t^2}}} = - \int \frac{\frac{dt}{t}}{\sqrt{1+t-t^2}} = - \int \frac{dt}{\sqrt{1-(t-\frac{1}{2})^2+\frac{1}{4}}} = \\ & \left\{ \begin{array}{l} t - \frac{1}{2} = z \\ dt = dz \end{array} \right\} \\ &= - \int \frac{dz}{\sqrt{\frac{5}{4}-z^2}} = - \arcsin \frac{2}{\sqrt{5}} z = - \arcsin \frac{2}{\sqrt{5}} (t - \frac{1}{2}) \end{aligned}$$

$$= - \arcsin \frac{2}{\sqrt{5}} (\frac{1}{x} - \frac{1}{2}) = - \arcsin \frac{2}{\sqrt{5}} (\frac{2-x}{2x}) = - \arcsin \frac{2-x}{x\sqrt{5}} + C$$

1270

$$\begin{aligned} \int \frac{dx}{(x-1)\sqrt{x^2-2}} & \left\{ \begin{array}{l} x-1 = t \\ dx = dt \end{array} \right\} \\ &= \int \frac{dt}{t\sqrt{(t+1)^2-2}} = \int \frac{dt}{t\sqrt{t^2+2t-1}} \\ &\left\{ \begin{array}{l} t = \frac{1}{z} \\ dt = -\frac{dz}{z^2} \end{array} \right\} \\ &= \int \frac{-\frac{dz}{z^2}}{\frac{1}{z}\sqrt{\frac{1}{z^2}+\frac{2}{z}-1}} = \int \frac{-\frac{dz}{z}}{\sqrt{\frac{1+2z-z^2}{z^2}}} = - \int \frac{dz}{\sqrt{1+2z-z^2}} = - \int \frac{dz}{\sqrt{2-(z-1)^2}} = - \arcsin \frac{z-1}{\sqrt{2}} \\ &= - \arcsin \frac{\frac{1}{t}-1}{\sqrt{2}} = - \arcsin \frac{\frac{1}{x}-1}{\sqrt{2}} = - \arcsin \frac{\frac{1-x+1}{x}}{\sqrt{2}} = - \arcsin \frac{\frac{2-x}{x}}{\sqrt{2}} \\ &= - \arcsin \frac{2-x}{\sqrt{2}(x-1)} + C \end{aligned}$$

1271

$$\begin{aligned} \int \frac{dx}{(x+1)\sqrt{x^2+2x}} &= \int \frac{dx}{(x+1)\sqrt{(x+1)^2-1}} \\ &\left\{ \begin{array}{l} x+1 = t \\ dx = dt \end{array} \right\} \\ &= \int \frac{dt}{t\sqrt{t^2-1}} \\ &\left\{ \begin{array}{l} t = \frac{1}{z} \\ dt = -\frac{dz}{z^2} \end{array} \right\} \\ &= - \int \frac{\frac{dz}{z^2}}{\frac{1}{z}\sqrt{(\frac{1}{z})^2-1}} = - \int \frac{\frac{dz}{z}}{\sqrt{(\frac{1}{z})^2-1}} = - \int \frac{\frac{dz}{z}}{\sqrt{\frac{1-z^2}{z^2}}} = - \int \frac{dz}{\sqrt{1-z^2}} = - \arcsin z \\ &= - \arcsin \frac{1}{t} = - \arcsin \frac{1}{x+1} + C \end{aligned}$$

1272

$$\begin{aligned} \int \sqrt{x^2+2x+5} dx &= \int \sqrt{x^2+2x+1+4} dx = \int \sqrt{(x+1)^2+4} dx \\ &\left\{ \begin{array}{l} x+1 = t \\ dx = dt \end{array} \right\} \\ &= \int \sqrt{t^2+4} dt \\ &\left\{ \begin{array}{l} t = 2 \sinh z \\ dt = 2 \cosh z dz \end{array} \right\} \\ &= \int \sqrt{4 \sinh^2 z + 4} 2 \cosh z dz = 4 \int \sqrt{\sinh^2 z + 1} \cosh z dz = 4 \int \cosh^2 z dz \\ &= 4 \int \frac{1}{2} (1 + \cosh 2z) dz = 2 \int (1 + \cosh 2z) dz = 2 \int dz + 2 \int \cosh 2z dz \end{aligned}$$

$$\begin{aligned} & \left\{ \begin{array}{l} 2z = u \\ 2dz = du \end{array} \right\} \\ & = 2z + \int \cosh u du = 2z + \sinh u = 2z + \sinh 2z \\ & = 2 \operatorname{arcsinh} \frac{t}{2} + \sinh(2 \operatorname{arcsinh} \frac{t}{2}) = 2 \operatorname{arcsinh} \frac{x+1}{2} + \sinh(2 \operatorname{arcsinh} \frac{x+1}{2}) + C \end{aligned}$$

1273

$$\begin{aligned} & \int \sqrt{x-x^2} dx = \int \sqrt{\frac{1}{4} - (x - \frac{1}{2})^2} dx \\ & \left\{ \begin{array}{l} x - \frac{1}{2} = t \\ dx = dt \end{array} \right\} \\ & \int \sqrt{\frac{1}{4} - t^2} dt = \int \frac{\frac{1}{4} - t^2}{\sqrt{\frac{1}{4} - t^2}} dt \\ & \int \frac{\frac{1}{4} - t^2}{\sqrt{\frac{1}{4} - t^2}} dt = (at + b) \sqrt{\frac{1}{4} - t^2} + \int \frac{c}{\sqrt{\frac{1}{4} - t^2}} dt' \\ & \frac{\frac{1}{4} - t^2}{\sqrt{\frac{1}{4} - t^2}} = a \sqrt{\frac{1}{4} - t^2} + \frac{-2t(at+b)}{2\sqrt{\frac{1}{4} - t^2}} + \frac{c}{\sqrt{\frac{1}{4} - t^2}} | \sqrt{\frac{1}{4} - t^2} \\ & \frac{1}{4} - t^2 = a(\frac{1}{4} - t^2) - t(at+b) + c \\ & \frac{1}{4} - t^2 = t^2(-2a) + t(-b) + c + \frac{1}{4}a \\ & \{a = \frac{1}{2}, b = 0\} \\ & .c + \frac{1}{4}a = \frac{1}{4} \dots c + \frac{1}{8} = \frac{1}{4} \\ & \{c = \frac{1}{8}\} \\ & I = \frac{t}{2} \sqrt{\frac{1}{4} - t^2} + \frac{1}{8} \int \frac{dt}{\sqrt{\frac{1}{4} - t^2}} = \frac{1}{4}t \sqrt{1-4t^2} + \frac{1}{8} \arcsin 2t \\ & = \frac{1}{4}(x - \frac{1}{2}) \sqrt{1-4(x - \frac{1}{2})^2} + \frac{1}{8} \arcsin 2(x - \frac{1}{2}) \\ & = \frac{1}{4}(2x-1) \sqrt{x-x^2} + \frac{1}{8} \arcsin(2x-1) + C \end{aligned}$$

1274

$$\begin{aligned} & \int \sqrt{2-x-x^2} dx = \int \sqrt{2 - \frac{1}{4} + \frac{1}{4} - x - x^2} dx = \int \sqrt{\frac{9}{4} - (x + \frac{1}{2})^2} \\ & \left\{ \begin{array}{l} x + \frac{1}{2} = t \\ dx = dt \end{array} \right\} \\ & = \int \sqrt{\frac{9}{4} - t^2} dt = \frac{1}{4}t \sqrt{9-4t^2} + \frac{9}{8} \arcsin \frac{2}{3}t \\ & = \frac{1}{4}(x + \frac{1}{2}) \sqrt{9-4(x + \frac{1}{2})^2} + \frac{9}{8} \arcsin \frac{2}{3}(x + \frac{1}{2}) \\ & = \frac{1}{2}x \sqrt{2-x^2-x} + \frac{1}{4} \sqrt{2-x^2-x} + \frac{9}{8} \arcsin \left( \frac{2}{3}x + \frac{1}{3} \right) + C \end{aligned}$$

1275

$$\begin{aligned} & \int \frac{xdx}{x^4-4x^2+3} = \int \frac{xdx}{x^4-4x^2+4-1} = \int \frac{xdx}{(x^2-2)^2-1} \\ & \left\{ \begin{array}{l} x^2-2=t \\ 2xdx=dt \end{array} \right\} \\ & = \frac{1}{2} \int \frac{dt}{t^2-1} = -\frac{1}{2} \operatorname{arctanh} t = -\frac{1}{2} \operatorname{arctanh}(x^2-2) + C \end{aligned}$$

1276

$$\begin{aligned} & \int \frac{\cos x}{\sin^2 x - 6 \sin x + 12} dx = \int \frac{\cos x}{(\sin x - 3)^2 + 3} dx \\ & \left\{ \begin{array}{l} \sin x - 3 = t \\ \cos x dx = dt \end{array} \right\} \\ & = \int \frac{dt}{t^2 + 3} = \frac{1}{\sqrt{3}} \arctan \frac{t}{\sqrt{3}} = \frac{1}{\sqrt{3}} \arctan \frac{\sin x - 3}{\sqrt{3}} + C \end{aligned}$$

1277

$$\begin{aligned} & \int \frac{e^x}{\sqrt{1+e^x+e^{2x}}} dx \\ & \left\{ \begin{array}{l} e^x = t \\ e^x dx = dt \end{array} \right\} \\ & = \int \frac{dt}{\sqrt{1+t+t^2}} = \int \frac{dt}{\sqrt{1+(t+\frac{1}{2})^2-\frac{1}{4}}} = \int \frac{dt}{\sqrt{\frac{3}{4}+(t+\frac{1}{2})^2}} = \operatorname{arcsinh} \frac{2t+1}{\sqrt{3}} = \operatorname{arcsinh} \frac{2e^x+1}{\sqrt{3}} + C \end{aligned}$$

1278

$$\begin{aligned} & \int \frac{\sin x dx}{\sqrt{\cos^2 x + 4 \cos x + 1}} = \int \frac{\sin x}{\sqrt{(\cos x + 2)^2 - 3}} dx = \\ & \left\{ \begin{array}{l} \cos x + 2 = t \\ -\sin x dx = dt \end{array} \right\} \\ & = - \int \frac{dt}{\sqrt{t^2-3}} = -\ln(t + \sqrt{t^2-3}) = -\ln(\cos x + 2 + \sqrt{\cos^2 x + 4 \cos x + 1}) + C \end{aligned}$$

1279

$$\begin{aligned} & I = \int \frac{\ln x dx}{x \sqrt{1-4 \ln x - \ln^2 x}} = \int \frac{\ln x dx}{x \sqrt{-(\ln x + 2)^2 + 4 + 1}} = \int \frac{\ln x dx}{x \sqrt{5 - (\ln x + 2)^2}} \\ & \left\{ \begin{array}{l} \ln x = u \\ \frac{dx}{x} = du \end{array} \right\} \\ & I = \int \frac{udu}{\sqrt{5-(u+2)^2}} \\ & \left\{ \begin{array}{l} u+2=z \\ du=dz \end{array} \right\} \\ & = \int \frac{z-2}{\sqrt{5-z^2}} dz = \int \frac{zdz}{\sqrt{5-z^2}} - 2 \int \frac{dz}{\sqrt{5-z^2}} \\ & I_1 = \int \frac{zdz}{\sqrt{5-z^2}} \left\{ \begin{array}{l} 5-z^2=t \\ -2zdz=dt \end{array} \right\} = \int \frac{dt}{\sqrt{t}} = -\sqrt{t} = -\sqrt{5-z^2} \\ & I_1 = -\sqrt{5-(u+2)^2} = -\sqrt{1-u^2-4u} = -\sqrt{1-\ln^2 x - 4 \ln x} \\ & I_2 = \int \frac{dz}{\sqrt{5-z^2}} = \arcsin \frac{z}{\sqrt{5}} = \arcsin \frac{u+2}{\sqrt{5}} = \arcsin \frac{\ln x + 2}{\sqrt{5}} \\ & I = I_1 - I_2 = -\sqrt{1-\ln^2 x - 4 \ln x} - 2 \arcsin \frac{\ln x + 2}{\sqrt{5}} + C \end{aligned}$$

1280

$$I = \int \frac{dx}{(x+a)(x+b)}$$





$$\begin{aligned}
x_{12} &= \frac{4\pm\sqrt{16-12}}{2} \\
x_1 &= \frac{4+2}{2} = 3 \\
x_2 &= \frac{4-2}{2} = 1 \\
\frac{1}{(x-3)(x-1)(x^2+4x+5)} &= \frac{a}{x-3} + \frac{b}{x-1} + \frac{cx+d}{x^2+4x+5} |(x-3)(x-1)(x^2+4x+5) \\
1 &= ax^3 + 3ax^2 + ax - 5a + bx^3 + bx^2 - 7bx - 15b + cx^3 - 4cx^2 + 3cx + dx^2 - 4dx + 3d \\
1 &= x^3(a+b+c) + x^2(3a+b-4c+d) + x(a-7b+3c-4d) - 5a - 15b + 3d \\
\left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 3 & 1 & -4 & 1 & 0 \\ 1 & -7 & 3 & -4 & 0 \\ -5 & -15 & 0 & 3 & 1 \end{array} \right] &\xrightarrow{-3} \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -2 & \frac{1}{2} & -1 & 0 \\ 0 & 2 & -1 & -\frac{3}{5} & -\frac{1}{5} \end{array} \right] : -5 \\
&= \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -2 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & -\frac{1}{2} & -1 - \frac{3}{5} & -\frac{1}{5} \end{array} \right] \\
&= \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & -2 & \frac{1}{2} & -1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{array} \right] \\
&= \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -7 - \frac{1}{2} & 2 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{array} \right] \\
&= \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -7 - \frac{1}{2} & 2 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{8}{5} & -\frac{1}{5} \end{array} \right] -15 \\
&= \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 2 & 0 \\ 0 & 0 & \frac{15}{2} & \frac{8*15}{5} & 3 \end{array} \right] \\
&= \left[ \begin{array}{ccccc} 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -7 & 1 & 0 \\ 0 & 0 & -\frac{15}{2} & 2 & 0 \\ 0 & 0 & 0 & 26 & 3 \end{array} \right] \\
\{d = \frac{3}{26}, b = -\frac{1}{20}, a = \frac{1}{52}, c = \frac{2}{65}\} \\
I &= \frac{1}{52} \int \frac{dx}{x-3} - \frac{1}{20} \int \frac{dx}{x-1} + \int \frac{\frac{2}{65}x + \frac{3}{26}}{x^2+4x+5} dx \\
I &= \frac{1}{52} \ln(x-3) - \frac{1}{20} \ln(x-1) + \frac{1}{65} \int \frac{2x + (\frac{3}{26})}{x^2+4x+5} dx
\end{aligned}$$

$$\begin{aligned}
I &= \frac{1}{52} \ln(x-3) - \frac{1}{20} \ln(x-1) + \frac{1}{65} \int \frac{2x + \frac{15}{2}}{x^2+4x+5} dx \\
I_1 &= \frac{1}{65} \int \frac{2x+4+\frac{7}{2}}{x^2+4x+5} = \frac{1}{65} \int \frac{2x+4}{x^2+4x+5} + \frac{7}{2} \cdot \frac{1}{65} \int \frac{dx}{x^2+4x+5} \\
&\left\{ \begin{array}{l} x^2+4x+5 = t \\ (2x+4)dx = dt \end{array} \right\} \left\{ \begin{array}{l} x+2 = z \\ dx = dz \end{array} \right\} \\
I_1 &= \frac{1}{65} \int \frac{dt}{t} + \frac{7}{2} \cdot \frac{1}{65} \int \frac{dx}{(x+2)^2+1} = \frac{1}{65} \ln t + \frac{7}{130} \int \frac{dz}{z^2+1} \\
I_1 &= \frac{1}{65} \ln t + \frac{7}{130} \arctan z = \frac{1}{65} \ln(x^2+4x+5) + \frac{7}{130} \arctan(x+2) \\
I &= \frac{1}{52} \ln(x-3) - \frac{1}{20} \ln(x-1) + \frac{1}{65} \ln(x^2+4x+5) + \frac{7}{130} \arctan(x+2) + C \\
1294 \\
\int \frac{dx}{x^3+1} &= \int \frac{dx}{(x+1)(x^2-x+1)} \\
\frac{1}{(x+1)(x^2-x+1)} &= \frac{a}{x+1} + \frac{bx+c}{x^2-x+1} |(x+1)(x^2-x+1) \\
1 &= ax^2 + bx^2 - ax + bx + cx + c + a \\
1 &= x^2(a+b) + x(-a+b+c) + c + a \\
a+b &= 0 \\
-a+b+c &= 0 \\
c+a &= 1 \\
\left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{array} \right] &= \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \end{array} \right] (-2) \\
&= \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & -4 & -2 \end{array} \right] = \left[ \begin{array}{cccc} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & -3 & -2 \end{array} \right] \\
\{b = -\frac{1}{3}, c = \frac{2}{3}, a = \frac{1}{3}\} \\
I &= \int \frac{\frac{1}{3}}{x+1} dx + \int \frac{-\frac{1}{3}x + \frac{2}{3}}{x^2-x+1} dx = \frac{1}{3} \ln(x+1) - \frac{1}{3} \int \frac{x-2}{x^2-x+1} \\
I &= \frac{1}{3} \ln(x+1) - \frac{1}{6} \int \frac{2x-4}{x^2-x+1} \\
I &= \frac{1}{3} \ln(x+1) - \frac{1}{6} \int \frac{2x-1}{x^2-x+1} + \frac{1}{2} \int \frac{dx}{x^2-x+1} \\
I &= \frac{1}{3} \ln(x+1) - \frac{1}{6} \ln(x^2-x+1) + \frac{1}{2} \int \frac{dx}{(x-\frac{1}{2})^2 - \frac{1}{4} + 1} \\
I &= \frac{1}{3} \ln(x+1) - \frac{1}{6} \ln(x^2-x+1) + \frac{1}{2} \int \frac{dx}{(x-\frac{1}{2})^2 + \frac{3}{4}} \\
I &= \frac{1}{3} \ln(x+1) - \frac{1}{6} \ln(x^2-x+1) + \frac{\sqrt{3}}{3} \arctan \frac{2x-1}{\sqrt{3}} + C \\
1295 \\
\int \frac{dx}{x^4+1} &= \int \frac{dx}{(x^2+1)^2-2x^2+1-1} = \int \frac{dx}{(x^2+1)^2-2x^2} = \int \frac{dx}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)} \\
&\frac{1}{(x^2-\sqrt{2}x+1)(x^2+\sqrt{2}x+1)} = \frac{ax+b}{(x^2-\sqrt{2}x+1)} + \frac{cx+d}{(x^2+\sqrt{2}x+1)} \\
1 &= (ax+b)(x^2 + \sqrt{2}x + 1) + (cx+d)(x^2 - \sqrt{2}x + 1) \\
1 &= ax^3 + ax + a\sqrt{2}x^2 + bx^2 + b + b\sqrt{2}x + cx^3 + cx - c\sqrt{2}x^2 + dx^2 + d - d\sqrt{2}x
\end{aligned}$$



1298

$$\begin{aligned}
I &= \int \frac{3x+5}{(x^2+2x+2)} dx = \int \frac{3x+5}{((x+1)^2+1)} dx = 3 \int \frac{x}{((x+1)^2+1)} dx + 5 \int \frac{dx}{((x+1)^2+1)} = 3 \int \frac{x}{((x+1)^2+1)} dx + \\
I &= 3 \int \frac{x+1-1}{((x+1)^2+1)} dx + 5 \arctan(x+1) = 3 \int \frac{x+1}{((x+1)^2+1)} dx - 3 \int \frac{dx}{((x+1)^2+1)} + 5 \arctan(x+1) \\
I &= 3 \int \frac{x+1}{((x+1)^2+1)} dx - 3 \arctan(x+1) + 5 \arctan(x+1) = 3 \int \frac{x+1}{((x+1)^2+1)} dx + 2 \arctan(x+1) \\
I_1 &= \left\{ \begin{array}{l} x+1=t \\ dx=dt \end{array} \right\} = 3 \int \frac{t}{t^2+1} dt = \left\{ \begin{array}{l} t^2+1=u \\ 2tdt=du \end{array} \right\} = 3 \int \frac{du}{2u} = \frac{3}{2} \ln u = \\
&\frac{3}{2} \ln(t^2+1) \\
I_1 &= \frac{3}{2} \ln((x+1)^2+1) + C \\
I &= \frac{3}{2} \ln((x+1)^2+1) + 2 \arctan(x+1) + C
\end{aligned}$$

1299

$$\begin{aligned}
\int \frac{dx}{(x+1)(x^2+x+1)^2} \\
----- \\
\frac{1}{(x+1)(x^2+x+1)^2} &= \frac{a}{x+1} + \frac{bx+c}{(x^2+x+1)^2} + \frac{dx+e}{(x^2+x+1)} |(x+1)(x^2+x+1)^2 \\
1 &= a(x^2+x+1)^2 + (bx+c)(x+1) + (dx+e)(x+1)(x^2+x+1) \\
1 &= ax^4 + 2ax^3 + 3ax^2 + 2ax + a + bx^2 + bx + cx + c + dx^4 + 2dx^3 + 2dx^2 + dx + ex^3 + 2e \\
x^2 + 2ex + e & \\
1 &= x^4(a+d) + x^3(2a+e+2d) + x^2(3a+b+2d+2e) + x(2a+b+c+d+2e) + a+c+ \\
a+d &= 0 \\
2a+e+2d &= 0 \\
3a+b+2d+2e &= 0 \\
2a+b+c+d+2e &= 0 \\
a+c+e &= 1
\end{aligned}$$

$$\left[ \begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 2 & 1 & 0 \\ 3 & 1 & 0 & 2 & 2 & 0 \\ 2 & 1 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 \end{array} \right] \rightarrow \begin{array}{c} \text{Gaussova} \\ \text{eliminacija} \end{array} \rightarrow \left[ \begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{array} \right]$$

$$\{e=0\} \{d=-1\} \{c=0\}$$

$$b-d+2e=0$$

$$b+1=0$$

$$\{b=-1\} \{a=1\}$$

-----

$$\begin{aligned}
\int \frac{dx}{(x+1)(x^2+x+1)^2} &= \int \frac{dx}{x+1} + \int \frac{-xdx}{(x^2+x+1)^2} + \int \frac{-xdx}{x^2+x+1} \\
&= \ln(x+1) - \int \frac{dx}{x^2+x+1} - \int \frac{dx}{(x^2+x+1)^2} \\
&= \ln(x+1) - \int \frac{dx}{x^2+x+1+\frac{1}{4}-\frac{1}{4}} - \int \frac{dx}{(x^2+x+1)^2} \\
I &= \ln(x+1) - I_1 - I_2
\end{aligned}$$

$$\begin{aligned}
I_1 &= \int \frac{dx}{x^2+x+1+\frac{1}{4}-\frac{1}{4}} = \int \frac{dx}{\left(x+\frac{1}{2}\right)^2+\frac{3}{4}} = \\
I_1 &= \left\{ \begin{array}{l} x+\frac{1}{2}=t \\ dx=dt \end{array} \right\} = \\
I_1 &= \int \frac{dt}{t^2+\frac{3}{4}} = \frac{2}{\sqrt{3}} \arctan\left(\frac{2}{\sqrt{3}}t\right) = \frac{2}{\sqrt{3}} \arctan\left(\frac{2}{\sqrt{3}}(x+\frac{1}{2})\right) = \frac{2}{\sqrt{3}} \arctan\left(\frac{2x+1}{\sqrt{3}}\right) \\
I_2 &= \int \frac{dx}{(x^2+x+1)^2} = \int \frac{dx}{\left(\left(x+\frac{1}{2}\right)^2+\frac{3}{4}\right)^2} \\
I_2 &= \left\{ \begin{array}{l} x+\frac{1}{2}=t \\ dx=dt \end{array} \right\} \\
I_2 &= \int \frac{dt}{\left(t^2+\frac{3}{4}\right)^2} \\
I_2 &= \left\{ \begin{array}{ll} u=\frac{1}{(t^2+\frac{3}{4})^2} & dt=dv \\ -\frac{256t}{(4t^2+3)^3}dt=du & t=v \end{array} \right\} \\
I_2 &= \frac{t}{(t^2+\frac{3}{4})^2} - \int \left[ -\frac{256t^2}{(4t^2+3)^3} \right] dt = \frac{t}{(t^2+\frac{3}{4})^2} + 64 \int \frac{4t^2+3-3}{(4t^2+3)^3} dt \\
I_2 &= \frac{t}{(t^2+\frac{3}{4})^2} + 64 \int \frac{dt}{(4t^2+3)^2} - 3 \int \frac{dt}{(t^2+\frac{3}{4})^3} = \frac{t}{(t^2+\frac{3}{4})^2} + \int \frac{dt}{(t^2+\frac{3}{4})^2} - 3 \int \frac{dt}{(t^2+\frac{3}{4})^3} \\
I_2 &= \frac{t}{(t^2+\frac{3}{4})^2} + I_2 - 3 \int \frac{dt}{(t^2+\frac{3}{4})^3} \dots \text{ odusta san}
\end{aligned}$$