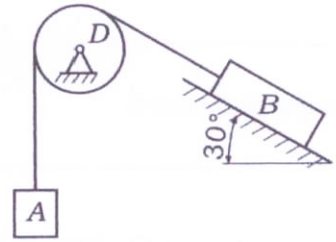
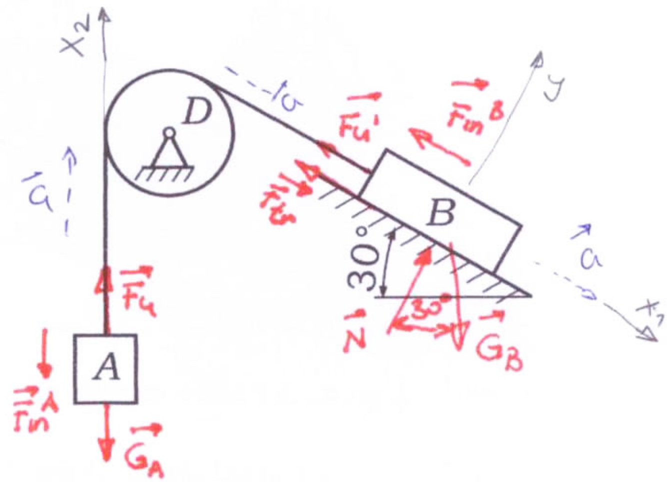
Tereti A i B spojeni su tankim nerastegljivim konopcem prebačenim preko kotura D. Pri spuštanju tereta B, mase $m_B = 4\ \text{kg}$, niz strmu ravan pod uglom $\alpha = 30°$, kotur D obrće se oko svoje nepomične ose, a teret A, mase $m_A = 1\ \text{kg}$, se podiže. Odrediti ubrzanje tereta A i B i silu u konopcu. Koeficijent trenja klizanja tereta B od strmu ravan je $\mu = 0{,}2$. Masu kotura D i konopca zanemariti.





Teret A

$$\vec{F_u} + \vec{G_A} + \vec{F_{in}^A} = 0$$

$$\vec{F_{in}^A} = -m\vec{a}$$

Projekcija na x_2:

$$F_u - F_{in}^A - G_A = 0$$

$$F_u - m_A a - m_A \cdot g = 0$$

$$F_u = m_A \cdot (a+g) \quad (*)$$

Teret B

$$\vec{F_u'} + \vec{F_{in}^B} + \vec{N} + \vec{G} = 0$$

$$\vec{F_{in}^B} = -m_B \cdot \vec{a}$$

$$F_u' = F_u$$

projekcija na y:

$$N - G_B \cdot \cos 30° = 0$$

$$N = m_B \cdot g \cos 30°$$

$$F_{tr} = N\mu = m_B \cdot g \mu \cdot \cos 30°$$

projekcija na x

$$G_B \sin 30° - F_u - F_t - F_B^{in} = 0$$

$$m_B \cdot g \cdot \sin 30° - F_u - m_B \cdot g \mu \cos 30° - m_B \cdot a = 0$$

$$F_u = m_B \left[g \cdot (\sin 30° - \mu \cdot \cos 30°) - a\right] \quad (**)$$

$$(*) = (**)$$

$$m_a \cdot (a+g) = m_B \left[g \cdot (\sin 30° - \mu \cdot \cos 30°) - a\right]$$

$$a = \frac{m_B \cdot g \cdot (\sin 30° - \mu \cdot \cos 30°) - m_A \cdot g}{m_A + m_B}$$

$$a = \frac{4 \cdot 9{,}81 \cdot (0{,}5 - 0{,}2 \cdot \frac{\sqrt{3}}{2}) - 1 \cdot 9{,}81}{1+4}$$

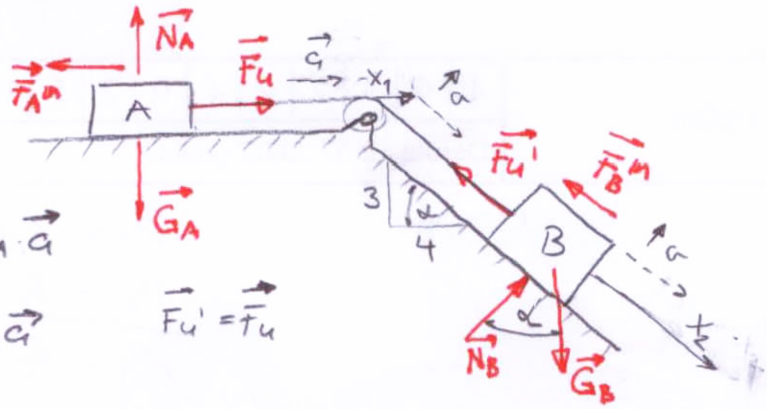
$$\boxed{a = 0{,}6027\ \frac{m}{s^2}}$$

iz (*):

$$F_u = 1 \cdot (0{,}6027 + 9{,}81)$$

$$\boxed{F_u = 10{,}413\ \text{N}}$$

Teret A mase 10 kg povezan je sa tankim nerastegljivim užetom sa teretom B mase 20 kg. Odrediti silu u konopcu. Sva trenja i masu kolura c zanemariti.



$\vec{F_A^{in}} = -m_A \cdot \vec{a}$

$\vec{F_B^{in}} = -m_B \cdot \vec{a}$ $\qquad \vec{F_u'} = \vec{F_u}$

Teret A

$\vec{F_u} + \vec{G_A} + \vec{N_A} + \vec{F_A^{in}} = 0$

x_1:

$F_u - F_A^{in} = 0$

$F_u = m_A \cdot a \quad (*)$

Teret B

$\vec{G_B} + \vec{N_B} + \vec{F_u'} + \vec{F_B^{in}} = 0$

x_2:

$G_B \cdot \sin\alpha - F_u - F_B^{in} = 0$

$\sin\alpha = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$

$F_u = m_B \cdot g \cdot \frac{3}{5} - m_B \cdot a \quad (**)$

$(*) = (**)$

$m_A \cdot a = m_B \cdot g \cdot \frac{3}{5} - m_B \cdot a$

$a = \dfrac{m_B \cdot g \cdot \frac{3}{5}}{m_A + m_B} = \dfrac{20 \cdot 9{,}81 \cdot \frac{3}{5}}{10 + 20}$

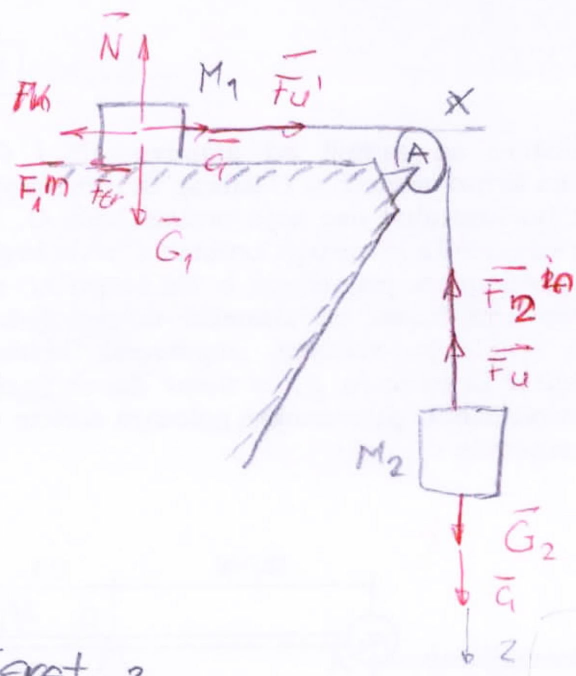
$\boxed{a = 3{,}924 \ \frac{m}{s^2}}$

12 (*)

$F_u = 10 \cdot 3{,}924$

$\boxed{F_u = 39{,}24 \ N}$

Teret M_1, težine G_1, nalazi se na hrapavoj horizontalnoj ravni. Za njega je privezano uže koje je zatim prebačeno preko kotura A i drugim krajem vezano za teret M_2, težine G_2. Zanemarujući težinu kotura i užeta, odrediti koeficijent trenja μ tereta M_1 o ravan, smatrajući ga konstantnim, ako se tereti kreću ubrzanjem a - istog intenziteta. Trenje užeta o kotur A zanemarit



Teret 1

$$\vec{F_u}' + \vec{N} + \vec{G} + \vec{F}_{tr} + \vec{F_1}^{in} = 0$$

$$F_1^{in} = \frac{G_1}{g} \cdot a$$

$$F_{tr} = N \cdot \mu = G_1 \mu$$

x:

$$F_u - G_1 \mu - \frac{G_1}{g} \cdot a = 0 \quad (1)$$

Teret 2

$$\vec{G_2} + \vec{F_u} + \vec{F_2}^{in} = 0$$

$$F_2^{in} = \frac{G_2}{g} \cdot a$$

z:

$$G_2 - F_u - \frac{G_2}{g} \cdot a = 0 \quad (2)$$

iz (1)

$$F_u = G_1 \cdot \mu + \frac{G_1}{g} \cdot a \quad (3)$$

iz (2)

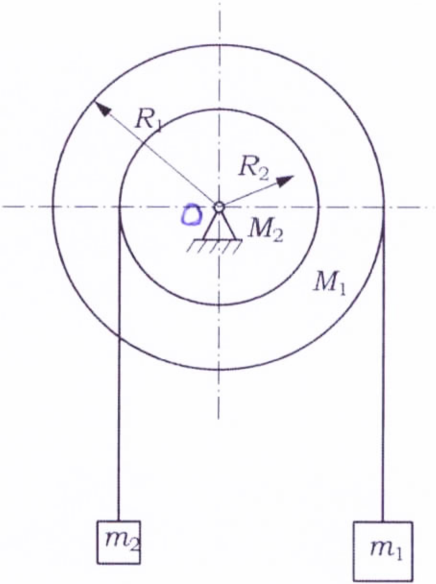
$$F_u = G_2 - \frac{G_2}{g} \cdot a \quad (4)$$

(3) = (4)

$$G_1 \mu + \frac{G_1}{g} a = G_2 - \frac{G_2}{g} \cdot a$$

$$\mu = \frac{G_2 - \frac{G_2}{g} \cdot a - \frac{G_1}{g} \cdot a}{G_1} = \boxed{\frac{G_2 g - a(G_2 + G_1)}{G_1 g}}$$

Dva tereta, masa m_1 i m_2 ($m_1=4m_2=m$), obješena su o dva laka nerastegljiva užeta, koji su obavijeni oko točkova poluprečnika R_1 i R_2, $R_1=\frac{3}{2}R_2$, masa $M_1=M_2=m/2$, prema slici. Točkovi su međusobno kruto spojeni i mogu se obrtati oko zajedničke horiznotalne ose O. Odrediti ugaono ubrzanje točkova smatrajući ih homogenim diskovima. Trenja zanemariti.



$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}}\right) - \frac{\partial E_k}{\partial q} = Q_q$$

1 stepen slobode - usvaja se generalisana koordinata φ pa su L j:

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) - \frac{\partial E_k}{\partial \varphi} = Q_\varphi \quad (1)$$

Kinetička energija sistema:

$$E_k = E_{k m_1} + E_{k m_2} + E_{k M_1} + E_{k M_2}$$

$$E_k = \frac{1}{2} m_1 (R_1 \dot{\varphi})^2 + \frac{1}{2} m_2 (R_2 \dot{\varphi})^2 + \frac{1}{2} \frac{M_1 R_1^2}{2} \dot{\varphi}^2 + \frac{1}{2} \frac{M_2 R_2^2}{2} \dot{\varphi}^2$$

$$m_1 = 4 m_2 = m \,; \quad M_1 = M_2 = \frac{m}{2} \,, \quad R_2 = \frac{2}{3} R_1$$

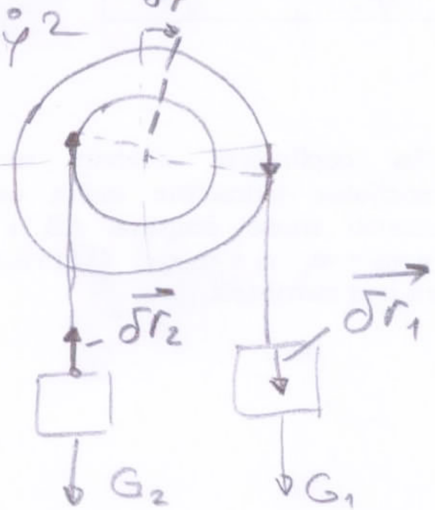
$$E_k = \frac{1}{2} m R_1^2 \dot{\varphi}^2 + \frac{1}{2} \cdot \frac{m}{4} \cdot \left(\frac{2}{3} R_1\right)^2 \dot{\varphi}^2 + \frac{1}{2} \cdot \frac{\frac{m}{2} R_1^2}{2} \cdot \dot{\varphi}^2 + \frac{1}{2} \frac{\frac{m}{2} \cdot \left(\frac{2}{3} R_1\right)^2}{2} \cdot \dot{\varphi}^2$$

$$= \frac{1}{2} m R_1^2 \dot{\varphi}^2 + \frac{1}{18} m R_1^2 \dot{\varphi}^2 + \frac{1}{8} m R_1^2 \dot{\varphi}^2 + \frac{1}{18} m R_1^2 \dot{\varphi}^2$$

$$= \frac{36+4+9+4}{72} m R_1^2 \dot{\varphi}^2 = \frac{53}{72} m R_1^2 \dot{\varphi}^2 \quad (2) \qquad R_1^2 = \frac{9}{4} R_2$$

$$\frac{53}{32} m R_2^2 \dot{\varphi}^2$$

Generalisana sila:



$$Q_\varphi = \frac{\delta A_\varphi}{\delta \varphi}$$

$$\delta A_\varphi = \vec{G}_1 \, \delta \vec{r}_1 + \vec{G}_2 \cdot \delta \vec{r}_2$$

(rad svih sila na elementarnom pomjeranju $\delta\varphi$)

$$\delta A_\varphi = m_1 g \cdot R_1 \delta\varphi + m_2 g \cdot R_2 \cdot \delta\varphi$$

$$\delta A_\varphi = m \cdot g R_1 \cdot \delta\varphi - \frac{m}{4} \cdot g \cdot \frac{2}{3} R_1 \delta\varphi$$

$$\delta A_\varphi = \frac{5}{6} m \cdot g \cdot R_1 \, d\varphi$$

$$Q_\varphi = \frac{5}{6} m \cdot g R_1 \quad (3) \qquad \frac{3}{2} \qquad \frac{5}{4} m g \cdot R_2$$

Ako sistemu dopustimo elementarno pomjeranje $\delta\varphi$, teret m_2 pomjeri se za $\delta r_2 = \delta\varphi \cdot R_2$, a teret m_1 za $\delta r_1 = \delta\varphi \cdot R_1$.

(3) i (2) u (1):

$$\frac{d}{dt}\left[\frac{\partial}{\partial\dot{\varphi}}\left(\frac{53}{72}mR_1^2\dot{\varphi}^2\right)\right]+\frac{\partial}{\partial\varphi}\left(\frac{53}{72}mR_1^2\dot{\varphi}^2\right)=\frac{5}{\cancel{6}}m\cdot g\cdot R_1$$

$$\frac{53}{36}mR_1^2\ddot{\varphi}+0=mg\cdot R_1$$

$$\ddot{\varphi}=\frac{\overset{6}{\cancel{36}}\cdot 5}{53\cdot\cancel{6}}\,\frac{g}{R_1}=\boxed{\frac{30}{53}\,\frac{g}{R_1}}$$

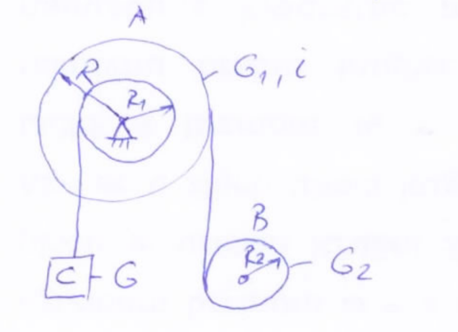
ili $\quad R_1=\frac{3}{2}R_2$

$$\boxed{\ddot{\varphi}=\frac{20}{53}\cdot\frac{g}{R_2}}$$

Sistem na slici se sastoji od doboša A težine G_1, poluprečnika inercije i za osu obrtanja, te diska B, težine G_2 i tereta C težine G. Kotur B počinje padati iz stanja mirovanja i pri tome obrće doboš A što izaziva podizanje tereta C. Odrediti ugaona ubrzanja ε_A i ε_B doboša i diska, ako je dato:

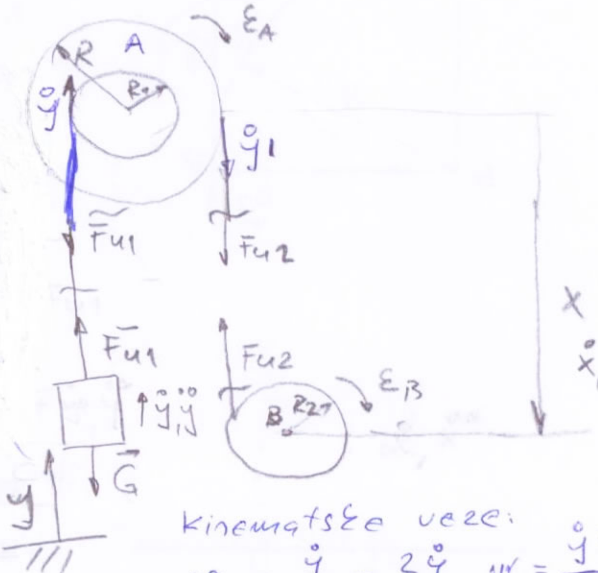
$R_1 = R_2 = R/2$ te $i = R/2$

Masu užeta i trenje zanemariti. [redacted]



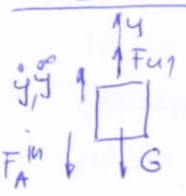
Rješenje:

Prvo rastavljamo sistem na podsisteme, postavljamo K. S



Postavljamo jednačine prema Dalamberovom principu za svaki podsistem:

Teret C



$F_A^{in} = \frac{G}{g} \cdot \ddot{y}$

Kinematske veze:

$\omega_A = \frac{\dot{y}}{R_1} = \frac{2\dot{y}}{R}$ $\quad \omega_A = \frac{\dot{y}'}{R}$

$\varepsilon_A = \frac{2\ddot{y}}{R}$ $\quad \dot{y}' = 2\dot{y}$ (A)

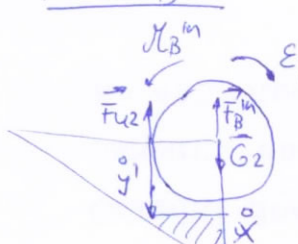
Translacija, po Dalamberovom principu je:

$\vec{G} + \vec{F}_{u1} + \vec{F}_A^{in} = 0$

projekcija na y

$F_{u1} - G - \frac{G}{g}\ddot{y} = 0$ (1)

Kotur B



$M_B^{in} = J_B \cdot \varepsilon_B$

$\bar{F}_B^{in} = \frac{G_2}{g} \ddot{x}$

$M_B^{in} = \frac{1}{2}\frac{G_2}{g} R_2^2 \cdot \frac{2(\ddot{x} - 2\ddot{y})}{R}$

$\omega_B = \frac{\dot{x} - \dot{y}'}{R_2} = \frac{2(\dot{x} - 2\dot{y})}{R}$; $\varepsilon_B = \frac{2(\ddot{x} - 2\ddot{y})}{R}$ (B)

Dal. princip:

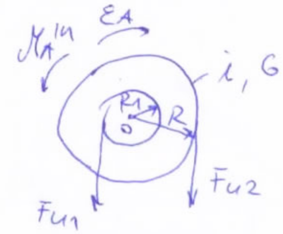
$\bar{G}_2 + \bar{F}_{u2} + \bar{F}_B^{in} = 0$

$F_{u2} \cdot R_2 - M_B^{in} = 0$

$G_2 - \frac{G_2}{g}\ddot{x} - F_{u2} = 0$ (2)

$F_{u2} \cdot \frac{R}{2} - \frac{G_2 R}{4g}(\ddot{x} - 2\ddot{y}) = 0$ (3)

Doboš A



suma rotacija:

$F_{u1} \cdot R_1 - F_{u2} \cdot R - M_A^{in} = 0$

$M_A^{in} = \frac{G_1}{g} i^2 \cdot \frac{2\ddot{y}}{R} = \frac{G_1}{g} \cdot \frac{R}{2}\ddot{y}$

$F_{u1} \cdot \frac{R}{2} - F_{u2} \cdot R + \frac{G_1}{g} \cdot \frac{R}{2}\ddot{y} = 0$ (4)

Jednačine 1, 2, 3, 4 – 4 nepoznate $F_{u1}, F_{u2}, \ddot{x}, \ddot{y}$

Iz (1) $F_{u1} = G - \frac{G}{g}\ddot{y}$ (5)

Iz (2) $F_{u2} = G_2 - \frac{G_2}{g}\ddot{x}$ (6)

Iz (3) $F_{u2} = \frac{G_2}{2g}(\ddot{x} - 2\ddot{y})$ (7)

Iz (7): $\ddot{x} = \frac{F_{u2} \cdot 2g}{G_2} + 2\ddot{y}$ (8)

(8) u (6)

$F_{u2} = \frac{G_2}{3} - \frac{2}{3}\frac{G_2}{g}\ddot{y}$ (9)

(6) = (7)

$\ddot{x} = \frac{2\ddot{y} + 2g}{3}$ (10)

(5) i (9) u (4)

$$\left(G + \frac{G}{g}\ddot{y}\right)\frac{R}{2} - \left(\frac{G_2}{3} - \frac{2}{3}\frac{G_2}{g}\ddot{y}\right)R + \frac{G_1}{g}\frac{R}{2}\ddot{y} = 0$$

$$\left(\frac{G}{2} - \frac{G_2}{3}\right)R + \frac{\ddot{y}}{g}R\left(\frac{G}{2} + \frac{2}{3}G_2 + \frac{G_1}{2}\right) = 0 \Big/ \cdot\frac{6}{R}$$

$$\ddot{y} = \frac{(2G_2 - 3G) \cdot g}{3G + 4G_2 + 3G_1} \quad (11)$$

(11) u (10)

$$\ddot{x} = \frac{2}{3}\frac{(2G_2 - 3G) \cdot g}{3G + 4G_2 + 3G_1} + \frac{2}{3}g\left(\frac{3G + 4G_2 + 3G_1}{3G + 4G_2 + 3G_1}\right)$$

$$\ddot{x} = \frac{\left(\frac{4}{3}G_2 - \cancel{2G} + \cancel{2G} + \frac{8}{3}G_2 + \frac{6}{3}G_1\right)g}{3G + 4G_2 + 3G_1} = \frac{(4G_2 + 2G_1)g}{3G + 4G_2 + 3G_1} \quad (12)$$

Iz (A)

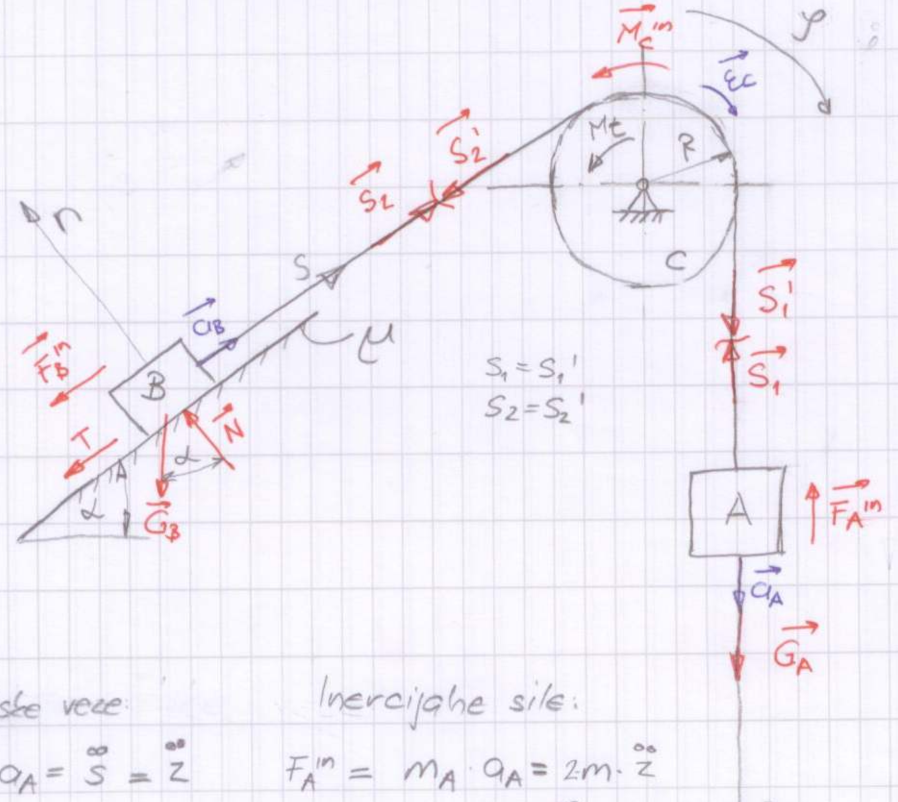
$$\varepsilon_A = \frac{2\ddot{y}}{R} = \frac{2}{R}\frac{(2G_2 - 3G) \cdot g}{3G + 4G_2 + 3G_1}$$

Iz (B)

$$\varepsilon_B = \frac{2}{R}(\ddot{x} - 2\ddot{y}) = \frac{2}{R}\left[\frac{(4G_2 + 2G_1)g - 2(2G_2 - 3G)g}{3G + 4G_2 + 3G_1}\right]$$

$$\varepsilon_B = \frac{2}{R}\frac{(2G_1 + 6G) \cdot g}{3G + 4G_2 + 3G_1} = \frac{4}{R}\frac{(G_1 + 6G) \cdot g}{3G + 4G_2 + 3G_1}$$

Preko homogenog koturа, mase $m_c = m$ i poluprečnika R prebačeno je neistegljivo uže zanemarljive mase koje je jednim krajem pričvršćeno za teret A, a drugim krajem za teret B. Masa tereta A je $m_A = 2m$. Teret B mase $m_B = m$ se kreće po strmoj ravni pod uglom $\alpha = 30°$ pri čemu je koeficijent trenja između tereta B i strme ravni $\mu = \frac{1}{\sqrt{3}}$. Pri obrtanju koturа C javlja se otpor u ležaju njegove osovine u iznosu $M_t = k \cdot \omega_c$, gdje je $k = \frac{mR^2}{4}$, a ω_c je ugaona brzina koturа C.
Ako je sistem u početnom trenutku mirovao odrediti brzinu tereta A, $v_A(t) = ?$



Kinematske veze:

$$a_B = a_A = \ddot{s} = \ddot{z}$$

$$\varepsilon_c = \ddot{\varphi} = \frac{\ddot{s}}{R} = \frac{\ddot{z}}{R}$$

$$\omega_c = \dot{\varphi} = \frac{\dot{z}}{R}$$

Inercijalne sile:

$$F_A^{in} = m_A \cdot a_A = 2m \cdot \ddot{z}$$

$$F_B^{in} = m_B \cdot a_B = m \cdot \ddot{z}$$

$$M_C^{in} = J_c \cdot \varepsilon_c = \frac{m_c R^2}{2} \cdot \frac{\ddot{z}}{R}$$

$$M_C^{in} = \frac{m R \ddot{z}}{2}$$

Teret A:

$\vec{G}_A + \vec{S}_1 + \vec{F}_A^{in} = 0$

z: $m_A g - S_1 - F_A^{in} = 0$

$2m \cdot g - S_1 - 2m \cdot \ddot{z} = 0$

$S_1 = 2mg - 2m \cdot \ddot{z} \quad \dots (1)$

Kotur C

$S_1 \cdot R - M_C^{in} - M_t - S_2 \cdot R = 0$

$S_1 \cdot R - \frac{mR\ddot{z}}{2} - k \cdot \omega_C - S_2 \cdot R = 0$

$S_1 \cancel{R} - \frac{mR \cdot \ddot{z}}{2} - \frac{m\cancel{R^2}}{4} \cdot \frac{\dot{z}}{\cancel{R}} - S_2 \cdot \cancel{R} = 0 \quad / : R$

$S_1 - S_2 - \frac{m}{2}\ddot{z} - \frac{m}{4}\dot{z} = 0 \quad \dots (3)$

Teret B:

$\vec{N} + \vec{S}_2 + \vec{F}_B^{in} + \vec{T} + \vec{G}_B = 0$

n: $N - m_B \cdot g \cdot \cos\alpha = 0$

s: $S_2 - m_B g \cdot \sin\alpha - F_B^{in} - T = 0$

$T = N \cdot \mu = m_B g \cdot \cos\alpha \cdot \mu$

$S_2 - m g \sin\alpha - m \cdot \ddot{z} - \mu \cdot m \cdot g \cos\alpha$

$S_2 = m \cdot (g \sin\alpha + \mu \cdot g \cdot \cos\alpha) + m\ddot{z} \quad \dots (2)$

(1) i (2) u (3):

$2mg - 2m \cdot \ddot{z} - mg(\sin\alpha + \mu \cdot \cos\alpha) - m\ddot{z} - \frac{m}{2}\ddot{z} - \frac{m}{4}\dot{z} = 0$

$\left(2 - \sin 30° - \frac{1}{\sqrt{3}}\cos 30°\right) \cdot mg - \frac{7}{2}m\ddot{z} - \frac{m}{4}\dot{z}$

$mg - \frac{7}{2}m\ddot{z} - \frac{m}{4}\dot{z} = 0 \quad / \cdot -\frac{2}{7}m$

$$\boxed{\ddot{z} + \frac{1}{14}\dot{z} = \frac{2}{7}g}$$

$$\boxed{\frac{d\dot{z}}{dt} + \frac{1}{14} \cdot \dot{z} = \frac{2}{7}g} \quad \dots (4)$$

Dif. jedn. 1. reda: Izvod iz matematike

$\frac{dy}{dx} + P(x) y = Q_x$

Rješenje u obliku:

$y(x) = e^{-\int P(x)dx} \left\{ \left[\int \left(e^{\int P(x)dx} \cdot Q(x)\right) dx \right] + C \right\} \quad \dots (5)$

Na osnovu (4) i (5) slijedi:

$$\dot{z} = e^{-\int \frac{1}{14} dt} \left\{ \left[\int \left(e^{\int \frac{1}{14} dt} \cdot \frac{2}{7} g \right) dt \right] + C \right\}$$

$$\int \frac{1}{14} dt = \frac{t}{14}$$

$$\int e^{t/14} \cdot \frac{2}{7} g \cdot dt = \left| \begin{matrix} \frac{t}{14} = ds \\ dt = 14\, ds \end{matrix} \right| = \frac{2}{7} g \cdot 14 ds \int e^{s} = 4g\, e^{\frac{t}{14}}$$

$$\dot{z} = e^{-\frac{t}{14}} \cdot \left(4g e^{\frac{t}{14}} + C \right) = 39{,}24 + C \cdot e^{-\frac{t}{14}}$$

Poč. uslovi: $t = 0 \quad \dot{z} = 0$

$$0 = 39{,}24 + C \cdot e^{0} \Rightarrow C = -39{,}24$$

$$\dot{z} = v_A = 39{,}24 \left(1 - e^{-\frac{t}{4}} \right) \quad \left[\frac{m}{s} \cdot s \right]$$