Univerzitet u Zenici
Mašinski fakultet
Predmet: **Dinamika**

Školska godina 2006/2007

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Pismeni ispit
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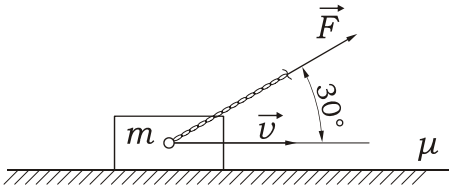
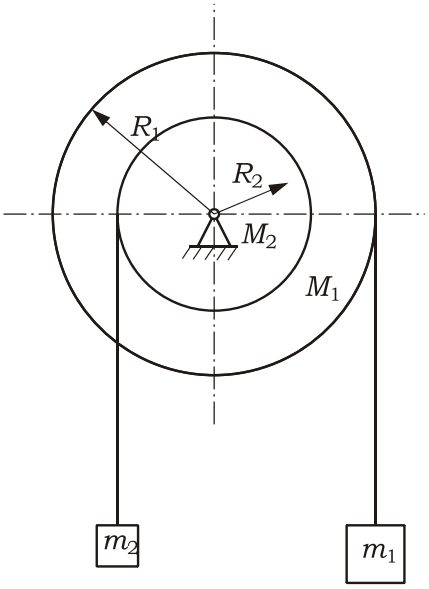
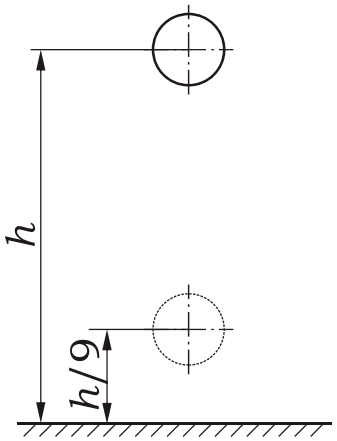
Zadaci:

1. Sanduk mase m = 400 kg vuče se po tlu pomoću sile konstantnog pravca i intenziteta F = 3 kN prema slici. Koeficijent trenja između sanduka i tla je μ = 0,4. Odrediti brzinu sanduka nakon pređenih 15 m pod dejstvom sile. Sila je počela djelovati na sanduk u stanju mirovanja.



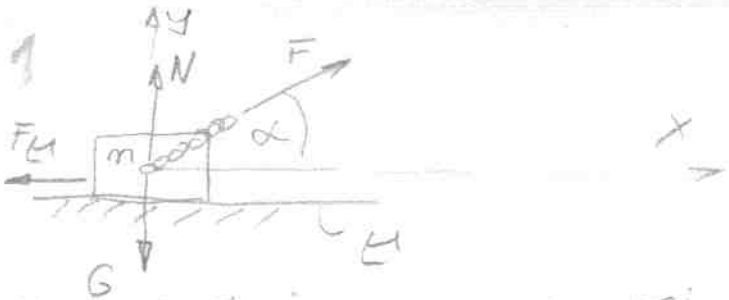
2. Dva tereta, masa m_1 i m_2 (m_1=4m_2=m), obješena su o dva laka nerastegljiva užeta, koji su obavijeni oko točkova poluprečnika R_1 i R_2, $R_1=\frac{3}{2}R_2$, masa M_1=M_2=m/2, prema slici. Točkovi su međusobno kruto spojeni i mogu se obrtati oko zajedničke horizontalne ose O. Odrediti ugaono ubrzanje točkova smatrajući ih homogenim diskovima. Trenja zanemariti.





3. Kuglica mase m pusti se sa visine h da padne na horizontalnu podlogu. Koeficijent sudara između kuglice i podloge je k = $\frac{1}{\sqrt{3}}$. Nakon koliko udara će se kuglica popeti na visinu jednaku h/9 uz pretpostavku da su svi udari normalni. Zanemariti otpor zraka.

1



Nema kretanja po y osi pa je

$\Sigma Y = 0$

$F \sin\alpha + N - G = 0$

$N = G - F \sin\alpha = mg - 3 \cdot 10^3 \cdot \sin 30° =$

$= 400 \cdot 9{,}81 - 3000 \cdot \frac{1}{2} =$

$= 2424\ N$

Projekcija na x osu, primjena zakona o promjeni kinetičke energije:

$\Delta E_{kx} = \Sigma A_{ix}$

$\frac{mv^2}{2} - \frac{mv_0^2}{2} = \int_0^{15} (F \cdot \cos\alpha - F_{tr})\, ds$

$= \int_0^{15} (F \cdot \cos 30° - N\mu)\, ds =$

$= \int_0^{15} \left(3 \cdot 10^3 \cdot \frac{\sqrt{3}}{2} - 2424 \cdot 0{,}4\right) ds$

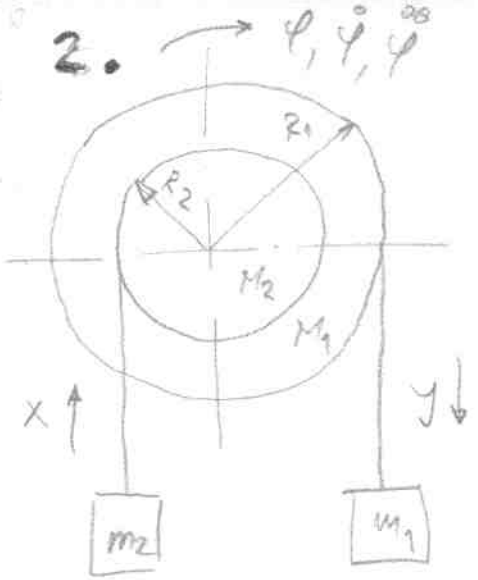
$= 1628{,}476 \cdot s\Big|_0^{15} = 1628{,}476 \cdot 15$

$\frac{mv^2}{2} = 24427{,}143\ Nm$

$v^2 = \frac{2 \cdot 24427{,}143}{m} = \frac{2 \cdot 24427{,}143}{400} = 122{,}136\ \frac{m^2}{s^2}$

$v = \sqrt{122{,}136} = \boxed{11{,}052\ \frac{m}{s}}$

2.



$m_1 = m$

$m_2 = \frac{m}{4}$

$M_1 = M_2 = \frac{m}{2}$

$R_1 = \frac{3}{2} R_2$

kinematske veze

$dy = R_1 \cdot d\varphi$ $\quad dx = R_2 \cdot d\varphi = \frac{2}{3} R_1 \cdot d\varphi$

$\dot{y} = R_1 \cdot \dot{\varphi}$ $\quad \dot{x} = R_2 \cdot \dot{\varphi} = \frac{2}{3} R_1 \cdot \dot{\varphi}$

$\ddot{y} = R_1 \cdot \ddot{\varphi}$ $\quad \ddot{x} = R_2 \cdot \ddot{\varphi} = \frac{2}{3} \cdot R_1 \cdot \ddot{\varphi}$

Zakon o promjeni kinet. energ.

$$\frac{dE_k}{dt} = \frac{dA}{dt} \quad (A)$$

$E_k = E_{k1} + E_{k2} + E_{k\,I} + E_{k\,II}$

$$E_{k1} = \frac{m_1 \dot{y}^2}{2} = \frac{m (R_1 \cdot \dot{\varphi})^2}{2} = \frac{m R_1^2 \dot{\varphi}^2}{2}$$

$$E_{k2} = \frac{m_2 \dot{x}^2}{2} = \frac{\frac{m}{4} \left(\frac{2}{3} R_1 \dot{\varphi}\right)^2}{2} = \frac{m R_1^2 \dot{\varphi}^2}{18}$$

$$E_{k\,I} = \frac{J_I \dot{\varphi}^2}{2} = \frac{1}{2} \left(\frac{M_1 R_1^2}{2}\right) \dot{\varphi}^2 = \frac{1}{2} \left(\frac{\frac{m}{2} R_1^2}{2}\right) \dot{\varphi}^2 = \frac{m R_1^2 \dot{\varphi}^2}{8}$$

$$E_{k\,II} = \frac{J_{II} \dot{\varphi}^2}{2} = \frac{1}{2} \left(\frac{M_2 R_2^2}{2}\right) \dot{\varphi}^2 = \frac{1}{2} \frac{\frac{m}{2} \left(\frac{2}{3} R_1\right)^2}{2} \dot{\varphi}^2 =$$

$$E_{k\,II} = \frac{m R_1^2 \dot{\varphi}^2}{18}$$

$$E_k = \left(\frac{mR_1^2}{2} + \frac{mR_1^2}{18} + \frac{mR_1^2}{8} + \frac{mR_1^2}{18}\right)\dot{\varphi}^2 =$$

$$= \frac{36+4+9+4}{72} mR_1^2\dot{\varphi}^2 = \frac{53}{72} mR_1^2\dot{\varphi}^2$$

$$\frac{dE_k}{dt} = \frac{106}{72} mR_1^2\,\dot{\varphi}\,\ddot{\varphi} = \frac{53}{36} mR_1^2\dot{\varphi}\ddot{\varphi} \quad (B)$$

$$dA = G_1\cdot dy - G_2\cdot dx = mg\cdot R_1\cdot d\varphi - \frac{m}{4}g\cdot\frac{2}{3}R_1 d\varphi =$$

$$= mgR_1 d\varphi - \frac{m}{6}gR_1 d\varphi = \frac{5}{6}mgR_1 d\varphi$$

$$\frac{d\varphi}{dt} = \frac{5}{6}mgR_1\dot{\varphi} \quad (C)$$

(B) i (C) u (A):

$$\frac{53}{36}mR_1^2\dot{\varphi}\ddot{\varphi} = \frac{5}{6}mgR_1\dot{\varphi}$$

$$\boxed{\ddot{\varphi} = \frac{36\cdot 5}{53\cdot 6}\cdot\frac{1}{R_1}g = \frac{30}{53}\frac{g}{R_1}}$$

3.

$$k = \frac{v'}{v}$$

$$v = \sqrt{2gh}$$

$$k = \frac{v_1}{v}\frac{\sqrt{2gh_1}}{\sqrt{2gh}} = \sqrt{\frac{h_1}{h}} \Rightarrow h_1 = k^2 h$$

$$h_2 = k^2 h_1 = k^2 k^2 h$$

$$h_n = k^{2n} h \Rightarrow \frac{h_n}{h} = k^{2n}$$

$$\ln\frac{h_n}{h} = \ln k^{2n}$$

$$\ln\frac{h_n}{h} = 2n \ln k$$

$$n = \frac{\ln\frac{h_n}{h}}{2\ln k}$$

$$n = \frac{\ln\frac{\frac{1}{9}h}{h}}{2\ln\frac{1}{\sqrt{3}}} = \frac{\ln\frac{1}{9}}{2\ln\frac{1}{\sqrt{3}}} = \frac{\ln 1 - \ln 9}{2(\ln 1 - \ln\sqrt{3})} = \frac{1}{2}\frac{\ln\sqrt{3}^4}{\ln\sqrt{3}} = \frac{1}{2}\frac{4\ln\sqrt{3}}{\ln\sqrt{3}}$$

$$n = 2$$