

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

$$a^{m+n} = a^m a^n$$

$$a^{mn} = (a^m)^n$$

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$a^{-m} = \frac{1}{a^m}$$

$$a^m b^m = (ab)^m$$

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$$

$$a^0 = 1$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln \frac{a}{b} = \ln a - \ln b$$

$$\ln a^b = b \ln a$$

$$\ln e = 1$$

$$\ln 1 = 0$$

$$\sin^2 \alpha + \cos^2 \alpha = 1$$

$$\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$$

$$\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$1 + \cos \alpha = 2 \cos^2 \frac{\alpha}{2}$$

$$1 - \cos \alpha = 2 \sin^2 \frac{\alpha}{2}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\operatorname{tg} x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x = \frac{1}{e}$$

$$\lim_{x \rightarrow 0} \left(1 + x\right)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow 0} \left(1 - x\right)^{\frac{1}{x}} = \frac{1}{e}$$

$$c' = 0$$

$$(x^a)' = ax^{a-1} (a \neq 0)$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$$

$$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$$

$$(a^x)' = a^x \ln a (a > 0)$$

$$(e^x)' = e^x$$

$$(\log_a x)' = \frac{1}{x \ln a} (a > 0, a = 1, x > 0)$$

$$(\ln x)' = \frac{1}{x} (x > 0)$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}} (|x| < 1)$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}} (|x| < 1)$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$$

$$\int dx = x + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C (n \neq -1)$$

$$\int \frac{1}{x} dx = \ln|x| + C (x \neq 0)$$

$$\int a^x dx = \frac{a^x}{\ln a} + C (a > 0, a \neq 1)$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \frac{1}{\cos^2 x} dx = \operatorname{tg} x + C$$

$$\int \frac{1}{\sin^2 x} dx = -\operatorname{ctg} x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}} dx = \ln|x + \sqrt{x^2 \pm a^2}| + C$$

Privr. f-je	ukupnih	prosečnih
prihoda	$P(p) = p \cdot x(p); P(x) = x \cdot p(x)$	$\overline{P}(x) = \frac{P(x)}{x}$
troškova	$C(x)$	$\overline{C}(x) = \frac{C(x)}{x}$
dobiti	$D(x) = P(x) - C(x)$	$\overline{D}(x) = \frac{D(x)}{x}$

Elastičnost funkcije: $y(x)$: $E_{y(x),x} = \frac{x}{y(x)} \cdot y'(x)$

Površina ravnog lika: $P = \int_a^b f(x) dx$, za $f(x) \geq 0$, $x \in [a, b]$	Zapremina obrtnog tela: $V = \pi \int_a^b f^2(x) dx$
Dužina luka krive: $l = \int_a^b \sqrt{1 + (f'(x))^2} dx$	Površina obrtnog tela: $P_o = 2 \pi \int_a^b f(x) \sqrt{1 + (f'(x))^2} dx$

Totalni diferencijal (opšta formula): $d^n f(x, y) = \left(\frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy \right)^n$

$$n = 1: df(x, y) = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$n = 2: d^2 f(x, y) = \frac{\partial^2 f}{\partial x^2} (dx)^2 + 2 \frac{\partial^2 f}{\partial x \partial y} dx dy + \frac{\partial^2 f}{\partial y^2} (dy)^2$$

$$n = 3: d^3 f(x, y) = \frac{\partial^3 f}{\partial x^3} (dx)^3 + 3 \frac{\partial^3 f}{\partial x^2 \partial y} (dx)^2 dy + 3 \frac{\partial^3 f}{\partial x \partial y^2} dx (dy)^2 + \frac{\partial^3 f}{\partial y^3} (dy)^3$$

Oblik	Tip ODJ	Smena
$y' = f(x) \cdot g(y)$	razdvaja promenljive	—
$y' = f\left(\frac{y}{x}\right)$	homogena	$u(x) = \frac{y(x)}{x}$
$y' = f\left(\frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}\right)$	svodi se na homogenu ($a_1b_2 - a_2b_1 \neq 0$)	$x = X + \alpha$, $y = Y + \beta$ $\alpha = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$, $\beta = \frac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$
$y' + f(x) \cdot y = g(x)$	linearna	$y = u(x) \cdot v(x)$
$y' + f(x) \cdot y = g(x) \cdot y^\alpha$	Bernulijeva ($\alpha \neq 0$)	$z(x) = y^{1-\alpha}(x)$

Homogena ODJ v.r. sa k.k.	Oblik rešenja
$y^{(n)} + a_{n-1}y^{(n-1)} + \dots + a_0y = 0$	<p>karakteristična jednačina (KJ): $r^n + a_{n-1}r^{n-1} + \dots + a_1r + a_0 = 0$</p> <p>(i) $r_i \in \mathbb{R}$ koren KJ višestrukosti $k_i \Rightarrow$</p> $y_i = C_1^i e^{r_i x} + C_2^i x e^{r_i x} + \dots + C_{k_i}^i x^{k_i-1} e^{r_i x}$ <p>(ii) $r_i = \alpha_i + \beta_i i$ koren KJ višestrukosti $k_i \Rightarrow$</p> $y_i = e^{\alpha_i x} (C_1^i \cos(\beta_i x) + C_2^i x \cos(\beta_i x) + \dots + C_{k_i}^i x^{k_i-1} \cos(\beta_i x))$ $+ e^{\alpha_i x} (D_1^i \sin(\beta_i x) + D_2^i x \sin(\beta_i x) + \dots + D_{k_i}^i x^{k_i-1} \sin(\beta_i x))$ <p>$y_h = y_1 + y_2 + \dots + y_l, i = 1, 2, \dots, l$ (l broj različitih korena KJ)</p>

Binomni obrazac: $(a+b)^n = \sum_{k=1}^n \binom{n}{k} a^k b^{n-k} = \sum_{k=1}^n \binom{n}{k} a^{n-k} b^k$					
$P_n = n!;$	$\bar{P}_n^{k_1, \dots, k_l} = \frac{n!}{k_1! \cdot \dots \cdot k_l!};$	$V_n^k = n(n-1) \cdot \dots \cdot (n-k+1);$	$\bar{V}_n^k = n^k;$	$C_n^k = \binom{n}{k}$	

SLOŽENI KAMATNI RAČUN

$$G_n = G \left(1 + \frac{p}{100}\right)^n; \quad G = \frac{G_n}{\left(1 + \frac{p}{100}\right)^n}; \quad n = \frac{\ln \frac{G_n}{G}}{\ln \left(1 + \frac{p}{100}\right)}; \quad p = \left(\sqrt[n]{\frac{G_n}{G}} - 1\right) \cdot 100$$

S.K.R. SA ČEŠĆIM KAPITALISANJEM

$$G_{mn} = G \left(1 + \frac{p}{100 \cdot m}\right)^{m \cdot n}; \quad G = \frac{G_{mn}}{\left(1 + \frac{p}{100 \cdot m}\right)^{m \cdot n}}; \quad n = \frac{\ln G_{mn} - \ln G}{m \cdot \ln \left(1 + \frac{p}{100 \cdot m}\right)}; \quad \text{neprecizna k.s.} = \frac{p}{m}$$

RAČUN ŠTEDNJE

$$S_n = U q \frac{q^n - 1}{q - 1}; \quad U = \frac{S_n (q - 1)}{q (q^n - 1)}; \quad n = \frac{\ln \left(1 + \frac{S_n (q - 1)}{U q}\right)}{\ln q}; \quad q = 1 + \frac{p}{100}$$

OTPLATA DUGA

$$R = D q^n \frac{q - 1}{q^n - 1}; \quad D = \frac{R (q^n - 1)}{q^n (q - 1)}; \quad n = \frac{\ln \frac{R}{D(q-1)}}{\ln q}; \quad q = 1 + \frac{p}{100}$$

$$\text{KONFORMNA KAMATNA STOPA} \quad \left(1 + \frac{p_k}{100}\right)^s = 1 + \frac{p}{100}; \quad p_k = \left(\sqrt[s]{1 + \frac{p}{100}} - 1\right) \cdot 100$$

Indeksi	formula	veza	Prosečna stopa rasta/pada
bazni	$B_i = \frac{y_i}{y_b} \cdot 100$	$B_i = \begin{cases} \frac{B_{i+1}}{V_{i+1}} \cdot 100, & i < b \\ 100\%, & i = b \\ \frac{B_{i-1}}{100} \cdot V_i, & i > b \end{cases}$	$y_i \nearrow \quad r_S = \left(\sqrt[n-1]{\frac{y_{\max}}{y_{\min}}} - 1\right) \cdot 100$
verižni	$V_i = \frac{y_i}{y_{i-1}} \cdot 100$	$V_i = \frac{B_i}{B_{i-1}} \cdot 100$	$y_i \searrow \quad p_S = \left(\sqrt[n-1]{\frac{y_{\min}}{y_{\max}}} - 1\right) \cdot 100$ inače $r_{PS} = \frac{1}{n-1} \sum_{i=2}^n (V_i - 100)$

Aritmetički niz	Geometrijski niz
$n \in \mathbb{N}$	$n \in \mathbb{N}$
$a_n = a_1 + (n-1)d$	$a_n = a_1 q^{n-1}$
$S_n = a_1 n + \frac{n(n-1)}{2} d$	$S_n = a_1 \frac{q^n - 1}{q - 1}$
$n = \frac{a_n - a_1}{d} + 1$	$n = \frac{\ln \left(1 + \frac{S_n \cdot (q-1)}{a_1}\right)}{\ln q}$

Vektori

$$\vec{a} = (a_1, a_2, a_3) = a_1 \vec{i} + a_2 \vec{j} + a_3 \vec{k} \quad \vec{b} = (b_1, b_2, b_3) = b_1 \vec{i} + b_2 \vec{j} + b_3 \vec{k} \quad \vec{c} = (c_1, c_2, c_3) = c_1 \vec{i} + c_2 \vec{j} + c_3 \vec{k}$$

$$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2} \quad \vec{a} \pm \vec{b} = (a_1 \pm b_1, a_2 \pm b_2, a_3 \pm b_3) \quad \alpha \vec{a} = (\alpha a_1, \alpha a_2, \alpha a_3), \alpha \in \mathbb{R}$$

Skalarni proizvod	Vektorski proizvod	Mešoviti proizvod
$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{b} \cdot \cos \angle(\vec{a}, \vec{b})$	$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$	$(\vec{a} \times \vec{b}) \cdot \vec{c} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$	$ \vec{a} \times \vec{b} = \vec{a} \cdot \vec{b} \cdot \sin \angle(\vec{a}, \vec{b})$	
$\begin{array}{c ccc} \cdot & \vec{i} & \vec{j} & \vec{k} \\ \hline \vec{i} & 1 & 0 & 0 \\ \vec{j} & 0 & 1 & 0 \\ \vec{k} & 0 & 0 & 1 \end{array}$	$\begin{array}{c ccc} \times & \vec{i} & \vec{j} & \vec{k} \\ \hline \vec{i} & 0 & \vec{k} & -\vec{j} \\ \vec{j} & -\vec{k} & 0 & \vec{i} \\ \vec{k} & \vec{j} & -\vec{i} & 0 \end{array}$	
$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a} \cdot \vec{b} = 0$	\vec{a}, \vec{b} su kolinearni $\Leftrightarrow \vec{a} \times \vec{b} = \vec{0}$	$\vec{a}, \vec{b}, \vec{c}$ su komplanarni $\Leftrightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = 0$
	Površina paralelograma nad \vec{a} i \vec{b} : $P = \vec{a} \times \vec{b} $	Zapremina paralelopipeda nad \vec{a}, \vec{b} i \vec{c} : $V = (\vec{a} \times \vec{b}) \cdot \vec{c} $

Ravan: T - proizvoljna tačka u ravni ($T(x, y, z)$)

\vec{r}_T - vektor položaja tačke T ($\vec{r}_T = \overrightarrow{OT} = (x, y, z)$)

Normalni vektorski oblik jednačine ravni: \vec{n} - vektor normale na ravan

$$\vec{n}_0 \text{ - jedinični vektor normale na ravan } (\vec{n}_0 = \frac{\vec{n}}{|\vec{n}|})$$

P - tačka prodora normale kroz ravan

$$p = |\vec{r}_P|$$

$$\vec{r}_T \vec{n}_0 - p = 0 \quad \text{ili} \quad \vec{r}_T \vec{n} - p|\vec{n}| = 0$$

Skalarni oblik jednačine ravni: $\vec{n} = A\vec{i} + B\vec{j} + C\vec{k}$

$$D = -p|\vec{n}| \quad (\Rightarrow p = -\frac{D}{\pm\sqrt{A^2+B^2+C^2}} \quad (p > 0))$$

$$Ax + By + Cz + D = 0$$

Segmentni oblik jednačine ravni: $A, B, C, D \neq 0$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Jednačina ravni kroz tačku $T_1(x_1, y_1, z_1)$ koja je normalna na vektor $\vec{n} = (A, B, C)$:

$$Ax + By + Cz - Ax_1 - By_1 - Cz_1 = 0$$

Jednačina ravni kroz tri tačke: $T_1(x_1, y_1, z_1), T_2(x_2, y_2, z_2), T_3(x_3, y_3, z_3)$:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Ugao između dve ravni α i β : $\vec{n}_\alpha = (A_1, B_1, C_1)$ - normala na α

$\vec{n}_\beta = (A_2, B_2, C_2)$ - normala na β

$$\cos \angle(\alpha, \beta) = \left| \frac{A_1 A_2 + B_1 B_2 + C_1 C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

$$\alpha \perp \beta \Leftrightarrow A_1 A_2 + B_1 B_2 + C_1 C_2 = 0$$

$\alpha \parallel \beta \Leftrightarrow \vec{n}_\alpha, \vec{n}_\beta$ kolinearni

Rastojanje tačke $T_1(x_1, y_1, z_1)$ od ravni α : $Ax + By + Cz + D = 0$:

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 + D}{\sqrt{A^2 + B^2 + C^2}} \right|$$

Prava (u \mathbb{R}^3): T - proizvoljna tačka na pravoj $(T(x, y, z))$

\vec{r}_T - vektor položaja tačke T ($\vec{r}_T = \overrightarrow{OT} = (x, y, z)$)

\vec{a} - vektor pravca prave ($\vec{a} = (a_1, a_2, a_3)$)

Vektorski oblik jednačine prave kroz tačku T_1 : $\overrightarrow{TT_1} = t\vec{a}, t \in \mathbb{R}$

$$\vec{r}_T = \vec{r}_{T_1} + t\vec{a}$$

Parametarske jednačine prave kroz tačku $T_1(x_1, y_1, z_1)$:

$$\begin{aligned} x &= x_1 + ta_1 \\ y &= y_1 + ta_2 \\ z &= z_1 + ta_3 \end{aligned}$$

Kanoničke jednačine prave kroz tačku $T_1(x_1, y_1, z_1)$:

$$\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$$

Jednačina prave kroz dve tačke $T_1(x_1, y_1, z_1)$ i $T_2(x_2, y_2, z_2)$:

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

Rastojanje tačke $T_2(x_2, y_2, z_2)$ od prave p : $\frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$: $T_1(x_1, y_1, z_1)$

$$d = \frac{|\overrightarrow{T_1 T_2} \times \vec{a}|}{|\vec{a}|}$$

Rastojanje između dve prave $p_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$ i $p_2 : \frac{x - x_2}{b_1} = \frac{y - y_2}{b_2} = \frac{z - z_2}{b_3}$:

$$p_1 \parallel p_2 \Rightarrow d(p_1, p_2) = d(T_1, p_2), T_1 \in p_1 \text{ proizvoljna}$$

$$p_1, p_2 \text{ mimoilazne} \Rightarrow d(p_1, p_2) = \sqrt{\frac{(a_2 - x_1)^2 + (a_3 - y_1)^2 + (a_1 - z_1)^2}{(a_2^2 + a_3^2 + a_1^2)(b_2^2 + b_3^2 + b_1^2)}}$$

Ugao između dve prave $p_1 : \frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$ i $p_2 : \frac{x - x_2}{b_1} = \frac{y - y_2}{b_2} = \frac{z - z_2}{b_3}$:

$$\cos \angle(p_1, p_2) = \frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2}}$$

Ugao između prave $p : \frac{x - x_1}{a_1} = \frac{y - y_1}{a_2} = \frac{z - z_1}{a_3}$ i ravni $\alpha : Ax + By + Cz + D = 0$:

$$\sin \angle(p, \alpha) = \cos \angle(\vec{a}, \vec{n}) = \left| \frac{Aa_1 + Ba_2 + Ca_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{A^2 + B^2 + C^2}} \right|$$